University of Maribor

Faculty of Natural Sciences and Mathematics

Doctoral Dissertation

DEVELOPMENT AND VALIDATION OF SPATIAL TRAINING CURRICULUM USING DYNAMIC GEOMETRY SOFTWARE FOR UNIVERSITY-LEVEL EDUCATION

May, 2020

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For my dear friends Tara and Sanda, who were there all the way.

Summary

Spatial abilities which are described by Linn and Petersen (1985) as “skill in representing, transforming, generating, and recalling symbolic, non-linguistic information“ (Linn & Petersen, 1985) are as such not trained nor they are taught in schools. They are left to develop naturally and on their, aided perhaps by some of the activities in childhood which have been shown to be predictors of highly developed spatial abilities, such as playing with construction toys, gender, math scores, previous experience in design-related courses such as drafting, mechanical drawing etc. (Sorby & Baartmans, 2000). Nevertheless, the importance of highly developed abilities for STEM (science, technology, engineering and mathematics) students has been scientifically proven (e.g. Gohm, Humphreys, & Yao, 1998; Humphreys, Lubinski, & Yao, 1993; Lohman, 1988, 1994a, 1994b; Smith, 1964), what directly implies that spatial abilities can have a significant impact on the success in not only studying STEM subjects, as well as success in STEM careers.

Throughout the years and worldwide researchers have recognised the value and correlation between success in STEM fields an highly developed spatial abilities and therefore attempted to develop exercises or whole experimental programmes for enhancement of spatial abilities (e.g. Sorby & Baartmans, 2000; Martín ‐ Dorta, Saorín & Contero, 2008; Güven & Kosa, 2008) with the aim of helping STEM students develop their spatial abilities, the success of which has been verified through positive correlation between spatial pre-tests and post-tests. This thesis outlines the developed experimental programme for spatial abilities enhancement.

The developed experimental programme employing GeoGebra 5 ran over the course of four weeks (1.5 hour per week) and was conducted at the Faculty of Science and Education in Mostar, Bosnia and Herzegovina. The sample consisted of first-year students, with 35 male and 17 female in experimental and 33 male and 19 female in control group. The programme content entailed following topics: 1) introduction to GeoGebra 5 and sketching basic geometrical objects in the software, 2) measuring wooden objects, sketching them on paper and then in GeoGebra and vice versa, 3) working with reflection and rotation in 2D using already designed GeoGebra apps, 4) solving various mathematical problems which require spatial visualisation by first sketching them on paper and then in GeoGebra.

For pre-testing Smith and Whetton (1988) spatial test has been used, atop of which a background questionnaire has been administered, which also contained items hypothesised as possible spatial abilities predictors. For post-test Smith and Whetton (1988) test has been administered once again, along with Newton and Bristoll (2009) test. Dependent variables were 1) students’ success during the experimental programme and 2) students’ results on the spatial abilities pre-test and post-test, whereas the independent variable was students' gender.

With the application of ANCOVA analysis it has been shown that average score was 55.89 for the experimental and 53.36 for control group ($F=54.572$, $df=1$, $p=0.000$, $\eta_p^2=0.406$), from what it can be concluded that differences between the control and experimental group at initial testing of spatial abilities are not statistically significant. Results also showed that with the experimental group a statistically significant jump in performance on the post-test has occurred ($t=-9.126$, $df=38$, $p<0.05$), whereas with regard to the control group no statistically significant changes in performance have been noted ($t=1.431$, $df= 43$, $p>0.05$). Similarly, statistically significant correlation between the scores of the control group on the initial and final Smith and Whetton (1988) test has also been found ($r=0.952$; $p<0.01$), as well as between the scores of the experimental group on the initial and final Smith and Whetton (1988), where a significant jump in performance has been noted ($r=0.833$; $p<0.01$). No statistically significant differences have been found either with regard to gender ($F=3.904$; $df=1$; $p=.052$) or between the experimental and control groups and gender ($F=1.688$; $df=1$; $p=.198$). Statistically significant correlations between the average mathematics grade in high school and any scores of any of the two groups has been found between the total score on the final Newton and Bristoll (2009) spatial test and average mathematics grade ($r=0.272$; $p<0.05$).

Keywords: *spatial abilities, spatial abilities enhancement, experimental programme, GeoGebra*

Table of Contents

Table of Figures

Table of Tables

1 Foreword

High school education worldwide is designed to give students broad education in a variety of subjects in order for them to be as prepared as possible for their university studies and hence focusing on a specific field. It has been attempted to take into consideration all factors which contribute to their readiness for their university studies and educational programmes keep being reviewed and tailored more to the needs of high school students for their future education.

Students who will enrol one of the universities focusing on science, technology, engineering and mathematics (in further text abbreviation STEM will be used) need to have certain abilities developed which will enable them to more easily understand and apply the university material which will be presented to them. Students who decide on enrolling one of universities falling into the STEM category for the most part show their inclination, interest and abilities pertaining to natural science already in high school, which are for the most part evident in high grades in mathematics, computer science and physics, which is a common conjecture when parents advise their children on their further course of education. There are however other abilities which fall into the equation and which are not assessed in high school or are addressed directly and one of them are spatial abilities.

Spatial abilities are honed through e.g. mathematical and physics problems but also in everyday life the students can find themselves in situations in which they need to employ their spatial abilities and through those situations also work on developing them. Perhaps they wish to redecorate their room and need to visualise the furniture on different positions and be able to visualise the look of the entire room by applying their spatial abilities in assessing the amount of space each piece of furniture is going to take and mentally rotating it. Perhaps they are parking their car and need to be perfectly aware of the dynamics of car movement and also space around them in order to make it fit into a certain spot. Or perhaps they are walking through a foreign town and need to know from which direction they came and be aware that the bridge they crossed some time ago is now behind their back because of the combination of left and right turns they took. A person who is not studying at a STEM university can find himself in one of these situations, but those with developed spatial abilities will be more equipped to solve them fast and in an efficient manner. And students with well-developed spatial abilities are better equipped for the challenges which the STEM universities will present them.

Their development is often left to a chance or is a side-effect of other practices, but spatial abilities as such are not directly trained or addressed. The correlation between well-developed spatial abilities and success at STEM universities has been shown by various researchers. Consequently, being such a valuable tool for their future success at university, it would be of great value to students to have the opportunity to develop and enhance their spatial abilities.

Researchers worldwide have used various tools for spatial skills training, from building blocks, paper and pencil method to geometry software and have shown that all of these various tools, if used within a carefully developed spatial skills training programme, enhance spatial abilities. The emphasis is placed on the specificity of the exercises employed, all of which when combined, result in an experimental programme which offers students a complete, step by step programme leading towards developing their spatial abilities, each to their individual extent, and thus also to more successful university studies.

This research aims at exactly this, in hope that the value of spatial abilities for not only STEM students but also in pre-university education in general, as preparation for university study, will be recognised, taken into account and addressed as an issue.

2 Definitions and different spatial skills models

Spatial ability refers to "skill in representing, transforming, generating, and recalling symbolic, non-linguistic information" (Linn & Petersen, 1985)

Myriad of different theories and definitions of spatial abilities components are offered by experts, but it must also be taken into account how they evolved throughout time given new research results and new findings, which contribute to the already existing models and either enrich them or add an entirely new dimension. In fact, factor structure of spatial ability has been a topic of active research since 1940s and is continuing since (Yilmaz, 2009). In this section those definitions which left an impact on the scientific world, but also those which are relevant for the topic of this thesis, will be outlined.

According to Piaget's theory, spatial visualisation abilities are acquired throughout three specific developmental stages. Children first learn topological spatial visualisation, i.e. being able to differentiate between objects' topological relationships with each other, what includes the concepts of closeness, location within a group and isolation. Second they acquire projective representation, i.e. the ability to visualise the appearance of a specific object from a different perspective. And thirdly the children learn combining the lastly acquired abilities with the concept of measurement (Sorby & Baartmans, 2000). His work also suggests that children's spatial ability reaches its final developmental phase at the age of twelve (Piaget & Inhelder, 1971). The authors also defined two types of spatial ability during the child's interaction with the environment. Perceptual spatial ability is being defined as the ability to perceive spatial relationships between objects, whereas conceptual spatial ability refers to the ability to conceive and manipulate a mental model of the environment.

The authors suggested that the three aforementioned stages through which children's spatial abilities development passes are preoperational, concrete operational and formal operational stages, respectfully. More specifically, in the preoperational stage, younger than six years old, the children's internal model is egocentric. They learn how to locate objects in their environment with respect to themselves and grasp limited topological spatial relationships, i.e. separateness, proximity etc. In the second, concrete operational stage, which occurs between ages seven to nine years old, they are developing a cognitive map with a reference frame, i.e. they are able to visualise view from some other point beside themselves, which allows them to visualise how an object would look from a different point. The children also begin to develop projective relations such as in front/behind, left/right, broadening up their spatial abilities to understand more complex topological relations. In

the last formal operational stage, which begins around the age of eleven, they develop a coordinate reference frame, i.e. mental network of locations in fixed positions relative to each other. Estimating relative distances through straight line and proportional scale reductions fall into this last stage and transition from projective to Euclidean space occurs (Piaget & Inhelder, 1971).

More recently, Huttenlocher and Newcombe (1999) suggest that spatial understanding develops earlier than Piaget suggested in his work. According to the authors infants six months old are able to use so called dead reckoning skills in order to understand the locations of objects around them, which is an inborn ability of understanding distances. Babies twelve months old are able to understand distances so that they are able to find hidden stimuli, whereas by eighteen months old they are able to understand navigate simple trajectories. Although Piaget proposed that children's ability to use fixed points in order to estimate distance and location does not develop until age nine or ten, Huttenlocher and Newcombe propose the age mark for this ability to be around two years old. The authors further propose that by three years old the children are able to utilise simplified maps and models and continue their mental development in spatial learning by the time they are nine or ten, given that they are encouraged to do so (Huttenlocher & Newcombe, 1999).

Spatial ability research evolved from research on intelligence. Whereas Spearman (1927) in the UK focused on intelligence as a single factor (Burt, 1949; Vernon, 1950), researchers in the US were more of the opinion that intelligence was composed of multiple factors (Thurstone, 1950; Cattell, 1971; Guilford, 1967). For the researchers it was difficult to distinguish spatial ability factors from intelligence because some of the factors are strongly connected to the concept of general intelligence (Mohler, 2008). According to Spearman (1904, 1927), intelligence is a reflected of "mental energy" or "intellectual capacity" which is present in a specific degree by every human being, is hereditary and cannot be changed through education. Apart from the general intelligence factor 'g', the author also presented specific intelligence factors ‘s’. Spearman’s two-factor intelligence model has been confirmed via factor analyses, although through subsequent research new factors have been discovered, which have been considered not exactly general but not completely specific either (Zarevski, 2000, p. 35-36).

In 1921 Thorndike published a paper in which he outlined important distinctions between three broader classes of intellectual functioning, arguing that, for that time standard, intelligence tests measured what he called ‘abstract’ intelligence. While agreeing that ‘abstract’ intelligence was just as important, he also outlined ‘mechanical’ and ‘social’ intelligence being equally important. His definition of ‘mechanical’ intelligence was as the ability to visualise relationships between objects and grasp how the physical world functioned. His definition of ‘mechanical’ intelligence is held for one of the foundations of future spatial ability research (Thorndike, 1921; according to Mohler, 2008).

Around the same time El Koussy (1935) examined spatial intelligence and stroke foundations for its measurement. He found evidence for what he called ‘factor K’ existence, which he defined as the ability to obtain and utilise spatial imagery. Kelley (1928) proposed that manipulation of spatial relations was another distinct factor within spatial abilities category. Few years later Thurstone (1938) defined a ‘space’ factor which he explained as the ability to perform mental operations on spatial or visual images. His new findings have led him to propose that intelligence consisted of several primary mental abilities and he was one of the first to propose and demonstrate these factors through his *Multiple Factors theory*. It identified seven primary mental abilities, comprising of: associative memory, number facility, perceptual speed, reasoning, spatial visualisation, verbal comprehension and word fluency. His new theory resulted in intelligence tests which yield a single score by adding scores awarded in several abilities assessment (Mohler, 2008).

Thurstone later identified three primary spatial factors within spatial ability (Thurstone, 1950) the names for which were more descriptively replaced by Smith (Smith, 1964) as follows: (S1) *Mental rotation* was defined as the ability to recognise an object if it has been moved or when viewed from a different angle, (S2) *spatial visualisation* as the ability to recognise the parts of an object if it has been moved from its position and (S3) *spatial perception* as the ability to use one's own body orientation when relating to questions regarding spatial orientation (Mohler, 2008).

Sometime later P. E. Vernon proposes a four-level empirical hierarchical model. At the sheer top he placed cognitive ability. On the second level he placed verbal-educational and spatial-mechanic ability. Verbal-educational factor he divided on the third level into verbal and numerical abilities, whereas spatial-mechanic ability he divided on the third level into spatial abilities and manual ability to use mechanical information (Vernon, 1950: according to Zarevski, 2000, p. 37). On level four he placed specific factors. Vernon's proposed model was partially verified by Reuchlin and Valin research (1953), who proposed three broad, second-level factors: perceptive reasoning, symbolic reasoning and educational factor. The aforementioned factors build together a general cognitive factor (Zarevski, 2000, p. 37).

French (1951) proposed a classification which included 59 intelligence factors, whereas in 1976 Ekstrom et al (1976) identified two components of spatial ability: spatial orientation and spatial visualisation. The authors defined spatial orientation as the ability to perceive spatial patterns or to maintain orientation with respect to objects in space, whereas they defined spatial visualisation as the ability to manipulate or transform the image of spatial patterns into other arrangements or the mental rotation of an object or objects in short term memory and performance of a series of operations. According to McGee (1979b), spatial orientation involves the comprehension of arrangement of elements within a visual stimulus pattern, aptitude for remaining unconfused by random orientations in which a configuration may be presented and the ability to determine spatial relations in which the body orientation of the observer is an essential part of the problem. Also according to McGee, spatial visualisation is an ability to mentally manipulate, rotate, twist or invert pictorially presented spatial visual stimuli. This ability involves a process of recognition, retention and recall of a configuration in which there is a movement among the internal parts of the configuration, or of an object manipulated in 3D space or the folding and unfolding of flat patterns (McGee, 1979a; according to Mohler, 2008).

Guilford proposed in 1967 a complex model of human intelligence in which he presents 120 independent hypothetical factors of cognitive abilities. Each of the factors has been assigned a unique three-letter label, where one side of the cube refers to operation, second to content and third to product. The model comprises of the following operations: cognition (C), convergent production (N), evaluation (E), divergent production (D) and memory (M). With regard to content, the model comprises of: figurative (F), symbolic (S), semantic (M) and behavioural (B). And lastly with regard to product the model comprises of: units (U), classes (C), relations (R), systems (S), transformations (T) and implications (I). The factors have been defined operationally, i.e. in terms of measurement techniques and the three-letter unique label places them on a specific location in the 3D space (Zarevski, 2000; p.40).

According to Linn and Petersen (1985), spatial ability refers to "skill in representing, transforming, generating and recalling symbolic, non-linguistic information" (Linn & Petersen, 1985). The authors outline three categories of spatial abilities: spatial perception, mental rotation and spatial visualisation. In their meta-study, focusing on gender differences with regard to spatial abilities categories, the authors outlined psychometric and cognitive perspective with regard to each category (Linn & Petersen, 1985).

J. B. Carroll, an American psychologist who was influenced by the work of L. Thurstone, reanalysed over 400 data sets of cognitive ability test scores and based on his conclusions in his book, "*Human Cognitive Abilities: A Survey of Factor Analytic Studies* (1993)", proposed a three-stratum model of human cognitive abilities, which suggests that the concept of intelligence can be portrayed as a hierarchical model of three strata. Stratum three of the general intellectual abilities is referred to the G factor, which accounts for the correlations among the broad abilities at Stratum two. Stratum two or broad abilities refers to eight broad abilities: fluid intelligence, crystallised intelligence, general memory and learning, broad visual perception, broad auditory perception, broad retrieval ability, broad cognitive speediness and processing speed. Stratum one or specific level refers to more specific factors grouped under the stratum two factors (Carroll, 1993). Spatial ability in his model lies in between verbal and mathematical abilities.

Furthermore, Carroll has conducted a comprehensive review of factor analytic studies of spatial ability and has proposed five main clusters: visualisation (Vz), Spatial Relations (SR), Closure Speed (CS), Flexibility of Closure (CF) and Perceptual Speed (P). Out of all these factors Carroll pointed out that the CF factor exists, "the psychometric evidence for the factor is somewhat ambiguous" (Carroll, 1993; p. 338). Carroll's three-strata model combines and brings together two very influential intelligence concepts: Spearman's g-factor model and Cattell's fluid and crystallised intelligence model. The factors on the second level categorise primary abilities into 8 broad domains: 1) fluid intelligence includes primary induction factors, reasoning, problem solving and visual perception factors, 2) crystallised intelligence focuses primarily on language and some of the factors categorised here are oral fluency, language development and comprehension of written text, 3) general memory and learning refers to associative memory, spontaneous retrieval, meaningful memory and memory span, 4) broad visual perception can be described through the primary factors of visualisation, spatial relations, mechanical knowledge and perceptual speed, 5) broad auditory perception includes factors such as speech perception and musical discrimination, 6) broad retrieval ability comprises of idea fluency, originality, expression fluency, associative fluency, figural fluency and problem sensitivity, 7) broad cognitive speed refers to perceptive speed, writing speed and numerical facility factor and 8) processing speed refers to the processing speed within elementary cognitive processes (Carroll, 1993).

Lohman (1988) proposes a model for spatial ability consisting out of three factors: spatial visualisation, spatial orientation and spatial relations. The author defines spatial visualisation as the ability to understand mental movements in a 3D space or to manipulate objects mentally. Spatial orientation is defined as the ability to track changes in the orientation of visual stimuli which requires mental rotation or configuration. Spatial relation is defined by the speed in manipulating simple visual patterns like mental rotation fast and correctly (Lohman, 1988). While Lohman acknowledges the existence of other factors, he argues that they are not central to the bulk of the concept of spatial abilities. He defines spatial visualisation as the ability to comprehend imaginary movements in a 3D space, i.e. to manipulate objects mentally. The author emphasises that this factor is defined by complex spatial tasks which require a sequence of transformations of a spatial representation, e.g. folding and unfolding a piece of paper which, when folded, has been perforated more than once. He defines spatial orientation as the ability to remain unconfused by the changing orientations in which the object may be presented. Essential nature of this factor is the awareness of whether the object is to the right or left, higher or lower, nearer or farther with regard to another.

In one of his later works, the author defines spatial ability as the "ability to generate, retain, retrieve and transform well-structured visual images" (1996). He points out that there are several spatial abilities, whereas each emphasises different aspects of the process of image creating, how it is stored, accessed and consequently transformed. Lohman argues that spatial abilities are "pivotal constructs of all models of human abilities". Hierarchical models place verbal-educational and spatial-visualisation factors just under the general ability since it is held that the verbal-spatial dimension requires more variance than any other dimension (Lohman, 1996).

With regard to the spatial orientation factor, Hegarty and Waller (2004) interpreted it as the ability to make egocentric spatial transformations in which one's one viewpoint changes with regard to the environment, whereas the relationship between object-based and environmental frames of reference does not change, e.g. visualising how an object would look from a different angle. Consequently, one of more recent tests developed by Kozhevnikov and Hegarty (2001) contains a task for the subjects in which it is required of them to imagine themselves facing a particular direction on a map or within an array. Spatial relations are more specifically defined as the ability to make object-based spatial transformations in which the positions of the objects are moved with regard to the environment, although one's own viewpoint does not change. Measurement of the development of this factor also often entails time-limited performance of 2D and 3D mental rotations (Hegarty & Waller, 2004).

	Spatial visualisation	Spatial orientation	Spatial relations
Definition	• ability to comprehend imaginary movements in a 3D space • ability to manipulate objects in imagination • defined by complicated spatial tasks which require a sequence of transformations of a spatial representation and more complex stimuli	• ability to keep track of the changing orientations in which a spatial configuration may be presented • estimating whether one object is to the right or left, higher or lower or nearer of farther than another • ability to make egocentric spatial transformations in which one's egocentric reference frame changes with respect to the environment, while the relation between object-based and environmental frames of reference does not change	• ability to mentally rotate a spatial object as a whole fast and correctly *(Colom et al., 2001).* • ability to make object based spatial transformations in which the positions of objects are moved with respect to an environmental frame of reference, but one's egocentric reference frame does not change
Example	imagine the folding and unfolding of a piece of paper which, when folded, has been perforated one or more times	imagine how a shape would appear from a different perspective and then to make a judgment from that imagined perspective	Spatial relations tests require speeded performance of 2D and 3D mental rotation items

Table 1 Lohman's model of spatial abilities (Lohman, 1979)

Apart from spatial ability factors and different models, specific terminology which is often employed when referring to the topic of spatial abilities, visualisation and visual imagery should be mentioned. Presmeg (1997) argues that *visualisation* includes processes of constructing and transforming both visual imagery and all spatial inscriptions which are a part of a mathematical activity. Roth (2004) explains the concept of *inscription* as graphical representations which are a focus of scientific practice, whereas according to Presmeg (1997) a *visual image* is a mental construct depicting visual or spatial information.

Mathematical visuality is described by Presmeg (1986) as the extent to which a person prefers visual methods of approach to mathematical problems to others, whereas *teaching visuality* is described as the extent to which a teacher uses visual presentation while teaching mathematics (Presmeg, 1986).

One other concept which is the focus of this dissertation is certainly spatial thinking. According to Newcombe (2010), *spatial thinking* concerns the locations of objects, their shapes, their relations to each other, and the paths they take as they move. A research done within the scope of project Talent in the US and in which 400 000 subjects participated showed that there is a high probability that subjects who had high scores on spatial tests in high school are going to study science, technology, engineering or mathematics, even after it was observed these subjects had high scores in verbal and mathematical skills (Newcombe, 2010).

And lastly, let us bring into connection with all these concepts also the concept of *software visualisation* and its application. There are many visualisation tools which have been developed by software designers for the sole purpose of exploring the software code, which use graphical representations for easier navigation, analysis and presentation of the information pertaining to software in order to enhance better understanding of it, and ergo enable the users to work with the software more quickly and efficiently in connection with this we also find the concept of *software exploration tools* which offer graphical representations of software structures with the aim of helping the user develop a mental model of the software. One of the most commonly used taxonomies employed to classify software visualisation tools has been written by Price, Baecker & Small (1993). This taxonomy is based on a generic model of a software visualisation tool. It employs the following categories: scope (range of software which can be visualised), content (visualised aspects), method (how the process of visualisation works), interaction (user's interaction with visualisation) and effectiveness (Storey, 2003: in Zhang, 2012).

3 Measuring spatial abilities and its factors

Throughout years of researching in this field, spatial abilities have been measured using four different types of tests: performance tests, paper-and-pencil tests, verbal tests, and dynamic tests on computers, whereas the performance tests have been the forerunners of all other types of tests, such as block manipulation or paper folding tasks which Binet and Simon (1916: according to Lohman, 1996) presented to children. Paper-and-pencil spatial abilities testing mostly consisted of several different types of items and were tested on young children of approximately the same age (Lohman, 1996).

To cover all spatial abilities tests in use worldwide would be beyond the scope of this work. However, several tests which have been used in studies relevant for the topic of this work will be outlined in the following sections.

3.1 Mental Rotation Test (MRT)

The MRT test requires subjects to compare pairs of 3D objects which are often rotated a certain angle and state if they are the same image or if one is a mirror image. The subjects are graded with regard to their ability to correctly and quickly identify mirror images. This test began its use in 1971 with Roger N. N. Shepard and Jacqueline Metzler, who conducted a chronometric research aiming at testing the mental rotation of 3D objects. Shepard hypothesised that this task would be executed by the participants by mentally rotating real objects and the results confirmed his hypothesis (Shepard & Metzler, 1971). This test has been redrawn and modernised since its first use and has been widely used by researchers as a tool for measuring spatial abilities level.

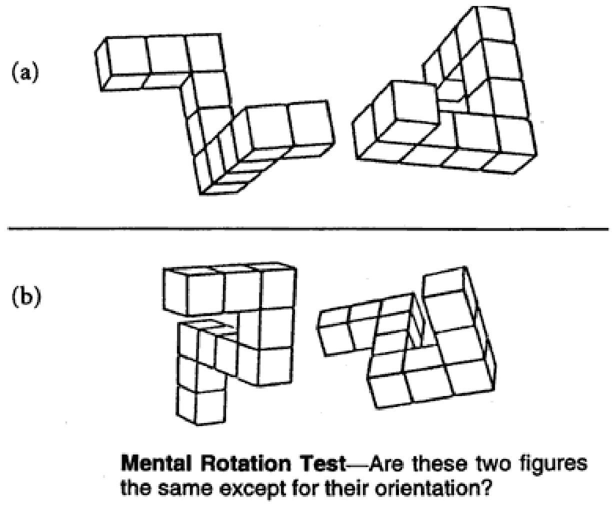

Figure 1. A sample from the MRT test.

Retrieved on the 14th of January, 2017 from http://mercercognitivepsychology.pbworks.com/w/page/61206659/Mental%20Rotation

3.2 The Differential Aptitude Test: Space Relations (DAT:SR)

The test itself consists of 50 items. It is required of the participants to choose the correct 3D object from four offered solutions which would result from folding the given 2D form.

In a 20-year longitudinal research done by Shea et al in 2001 among intellectually talented adolescents DAT:SR was one of the tests used, apart from using the DAT:MR (Mechanical Reasoning). It has been found that those participants whose favourite subject at school were math/science and who had stronger spatial abilities when compared to their verbal abilities were more likely to be found in engineering, computer science and mathematics occupations. Hence the authors concluded that spatial ability provides information for predicting the educational-vocational tracks which the students chose themselves. One drawback of this research in this particular aspect is that the sample consisted of intellectually talented individuals (Shea et al, 2001).

Gorska and Sorby (2008) administered during the school year 2005/06 an adapted version of the DAT:SR among students of middle and high school levels in the US. Average scores on this adapted version of the test were 50.4% for middle school students and 60.2% for high school students, with no gender differences observed for either schooling level and the Cronbach alpha indicated a reasonable test reliability (Gorska & Sorby, 2008).

3.3 Purdue Spatial Visualisation Test; Rotations (PSVT:R)

The original PSVT test consisted of three subtests: Developments, Rotations and Views and has been developed by Guay in 1976. It consisted of 36 items with 12 in each subtest. The PSVT:R is an extended version of the subtest Rotations, consists of 30 items with 13 symmetrical and 17 non-symmetrical pictures of 3D objects which are drawn in a 2D format (Maeda & Yoon, 2013).

In this test an object is shown which is then rotated in space for a certain amount. Second objects shown and it is required of the participant to mentally rotate this second object by exactly the same amount as the first object and then choose the correct picture (Sorby et al, 2014). The PSVT:R is predominantly used in educational research, particularly for the research in STEM disciplines, while MRT (Mental Rotation Test) has been often used for research in psychology and social studies. The popularity of use of the PSVT:R is according to Maeda and Yoon (2013) due to the following reasons: 1) research has shown strong reliability of the test and the valid evidence to support its use, 2) the ability required to solve the tasks outlined in the PSVT:R correspond to the abilities required to solve tasks often found in STEM disciplines (Bodner & Guay 1997; Yue 2004: according to Maeda & Yoon, 2013). 3) the test contains certain items which are by its difficulty enough to serve as a differentiation measure to distinguish between STEM students and students of other disciplines by their level of mental rotation ability (Black 2005; Yue 2006, according to: Maeda & Yoon, 2013), 4) the test has been cited as the most useful tool for measuring mental rotation ability which the most incorporates holistic or gestalt spatial thinking process and the least the analytic or analogical spatial thinking (Black 2005; Branoff 1998; Guay et al. 1978: according to Maeda & Yoon, 2013). 5) the test is available to researches for free and it is easy to score it because of its multiple choice format (Maeda & Yoon, 2013).

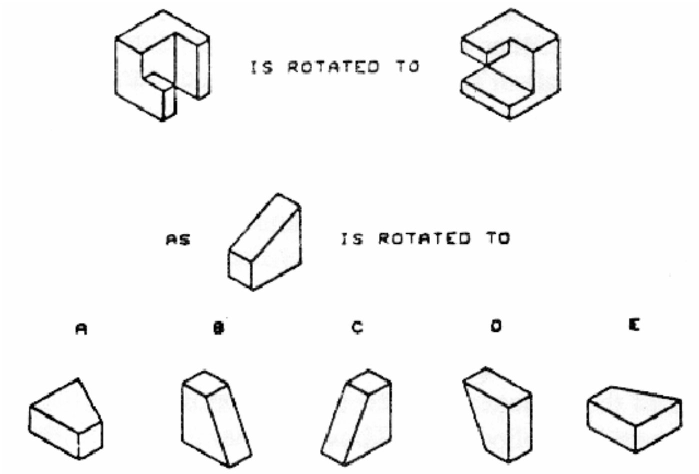

Figure 2. A sample from the PSVT:R test.

Retrieved on the 14th of January, 2017 from https://www.researchgate.net/figure/268982370_fig4_Figure-4-Purdue-Spatial-Visualization-Test-Rotations-PSVTR-example-problem-Guay

3.4 Mental Cutting Test (MCT)

This test measures the ability to visualise a cross-section of an object which has been cut by an imaginary plane. The participant is presented with an object and a plane which should be used for cutting on the left side and then needs to choose the correct cutting outcome out of five offered solutions (Sorby et al, 2014). The test itself consists of 25 problems with the maximum score of 25 points. In the test there are two types of problems: pattern recognition problems and quantity problems. Problems falling into the first category consist of different alternatives of possible cross sections, whereas the participant can come to the right solution by recognising the pattern of the section of the object and problems falling into the second category consist of possible solutions the cross sections of which are all similar and therefore in order to find the right solution one needs to mentally compare length, ratios and angles. Objects used in this test have complicated and unusual forms (Németh & Hoffmann, 2006), which is why it is difficult to geometrically recognise the solids even if they have been recognised topologically (Tsutsumi, 2004).

In the standard version of the test, after recognising and reviewing the constructional elements of the solid such as the vertex, edges and plane, the subject needs to recognise the relative location of the plane which will be used for cutting. In his research, Tsutsumi et al (1999) observed that subjects which had low scores did not seem to be able to correctly recognise the solid in the test and its plane and were consequently unable to visualise the correct solution. The researchers tried to make the subjects make a sketch which might help them in this task and offer the proposition that if intersecting lines were added to the picture of the test solid, that it might help the subjects to come to the correct solution (Tsutsumi, 2004).

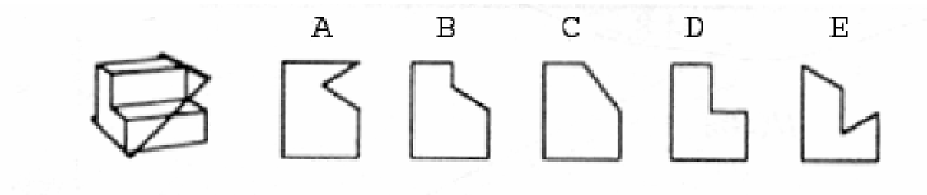

Figure 3. A sample from the MCT test.

Retrieved on the 14th of January, 2017 from https://www.researchgate.net/figure/268982370_fig1_Figure-1-Mental-Cutting-Test-MCT-example-problem-CEEB-1939

3.5 Spatial Test EG by I. MacFarlane Smith

This test has been published by the National Foundation for Educational Research in England and Wales (N.F.E.R.) and it deals with 2D material. It consists of six sub-tests, whereas each sub test is preceded by a practice test. The sub-tests are: fitting shapes, form recognition, pattern recognition, shape recognition, comparisons and form reflections. The maximum time given for its completion is cca one hour (Lean & Clements, 1981).

3.6 Spatial Test II by A. F. Watts, D.A. Pidgeon and M.K.B. Richards

This test has also been published by the N.F.E.R. and it deals with 3D material. It consists of five sub-tests, preceded by a practice test. The sub-tests are: matchbox corners, shapes and models, square completion, paper folding and block building. The maximum given time for its completion is cca 45 minutes (Lean & Clements, 1981).

3.7 Gestalt Completion Test by R.F. Street

It was published in 1931 by Teachers' College, Columbia University, New York, USA. It consists of 12 items, each of which is a black and white picture on which certain parts have been deleted. Incomplete pictures are then presented like a slide-film projected on a screen and it is required of subjects to complete the pictures mentally but also to describe in writing what they thought the pictures represented. First two items shown are practice examples. The time each item was exposed was about 10 seconds and the maximum time given for the test to be completed was cca 5 minutes (Lean & Clements, 1981).

3.8 3D Drawing Test by M.C. Mitchelmore

It was published in 1974 and consists of four separate tasks. It is required of subjects to draw parallel lines in space in order to represent 3D objects two-dimensionally. In the first exercise a diagram of a winding road is given with two light poles in the front and it is required of subjects to draw more poles alongside of the road, which task takes about 3 minutes. In the second exercise a half full bottle is shown and also how to represent the liquid inside. It is then required of the subjects to draw liquid surfaces on the example of bottles in various orientations. This task takes about 2 minutes. The third task consists of a cuboid made of many small wooden cubes, along with a diagram showing the cuboid. The subjects then need to complete drawings of other four blocks with no models shown to make them appear as though they have been constructed of small cubes. Seven minutes was allowed for this exercise. In the fourth and final exercise, a transparent plastic cube is shown along with a diagram representing the cube, using the dotted lines to portray the 'invisible lines'. It is then required of the subjects to complete diagrams of four prisms with no models shown, by adding the dotted lines. The time allowed for this exercise is 6 minutes (Lean & Clements, 1981).

3.9 Spatial ability Test (SAT) by Ekstrom et al (1976)

The SAT consists of two tests: Spatial Orientation Ability Test (SOAT) (Kayhan, 2005), which in turn consists of two subtests, Card Rotation Test (CRT) and Cube Comparison Test (CCT). CRT measures the ability to distinguish between the shapes and true-false items and consists of 169 items. CCT measures the ability of mental rotation and consists of tasks featuring six cubes, the sides of which are labelled with different numbers or letters. It is required of the subjects to decide whether the presented cubes are same or not. The CCT consists of 42 items.

Total score on the SOAT are the added scores on the CRT and CCT. Second test is Spatial Visualisation Test (SVAT) (Kayhan, 2005), which also consists of two sub-tests: Paper Folding Test (PFT) and Surface Development Test (SDT). The PFT consists of multiple choice items which require mental folding and unfolding a piece of paper, with 60 items present in the test. The SDT test requires visualising the development of different objects by folding paper and consists of 60 items (Turgut & Yilmaz, 2012).

4 Spatial abilities and STEM

"There is good evidence that [spatial ability] relates to specialized achievements in fields such as architecture, dentistry, engineering, and medicine . . . . Given this plus the longstanding anecdotal evidence on the role of visualization in scientific discovery . . . it is incredible that there has been so little programmatic research on admissions testing in this domain." (Richard E. Snow, 1999, p. 136)

Researchers worldwide have shown correlation between success in STEM and well-developed spatial abilities and as such, high scores on spatial abilities tests act as predictors for success in the field of STEM. However, as it has been noted before, spatial abilities as such are not directly trained or taught and for the most part it is relied on having them naturally developed, despite of plenty of evidence for the educational-occupational significance of spatial ability (Gohm, Humphreys, & Yao, 1998; Humphreys, Lubinski, & Yao, 1993; Lohman, 1988, 1994a, 1994b; Smith, 1964)

In a paper written in 1957, Super and Bachrach characterised spatial ability as an individual differences attribute with special importance for learning advanced scientific-technical material required for success in STEM. Apart from this, the authors also emphasised that attributes beyond spatial abilities, such as mathematical abilities, interests and non-intellectual attributes such as persistence, need as well to be studied. They were of the opinion that there was a great need for longitudinal studies which would follow development from early age and extending over 10 or 15 years (Super & Bachrach, 1957, p. 87).

Smith (1964) argues that there are at least 84 different careers in which spatial skills are important (Smith, 1964), whereas Maier argues that for technical professions such as engineering spatial visualisation skills and mental rotation skills are especially important (Maier, 1994).

Spatial abilities are considered to be closely related to success in mathematics and geometry specifically (Holzinger & Swineford, 1946). With regard to the field of mathematics specifically, it has been shown by Friedman (1995) in meta-analysis which included 75 studies that correlations between spatial and mathematical abilities ranged between 0.3 and 0.45 (Friedman, 1995). Guay and McDaniel showed that among elementary school children, those with high mathematics scores have greater spatial abilities unlike their peers with low mathematics scores (Guay & McDaniel, 1977, p. 214). Apart from this study, many other researchers reported positive relation of high spatial abilities to mathematics achievement (Battista, 1990; Fennema & Sherman, 1977; Kayhan, 2005; Turgut, 2007) but also to physics and science in general (Delialioğlu 1996; Delialioğlu & Aşkar, 1999). Furthermore, Hegarty and Waller supported the view that spatial abilities are important for constructing and understanding abstract spatial configurations in mathematical problem solving and emphasised that mathematical thinking required abilities associated with visual perception and spatial ability (Hegarty & Waller, 2005), whereas researchers emphasise the importance of spatial abilities for the sheer process of development of mathematical thinking (Gutiérrez, 1996; Presmeg, 2006).

4.1 Moses research and 'degree of visuality'

A pioneering research conducted by Moses in 1977 involved 145 fifth-grade elementary school students from the US, who were given a battery of six tests. Five of these tests were spatial abilities tests: Punched Holes, Card Rotations, Form Boards, Figure Rotations and Cube Comparisons. Sixth test comprised of a problem-solving inventory, i.e. ten non-routine mathematical problems. At the end of testing a threefold score was calculated: spatial ability score based on the z-scores from the five spatial tests, a problem-solving inventory from the sixth test and also a 'degree of visuality' score based on the number of e.g. pictures, graphs, lists or tables which the participants presented in their problem solutions in the problem-solving inventory. Although the correlations of spatial ability with problem-solving score and the 'degree of visuality' were significantly different than zero, the correlation between problem-solving performance and 'degree of visuality' was not significantly different than zero and thus the author concluded that spatial ability is a good predictor of problem-solving performance. Despite of the fact that subjects with high spatial abilities score high on pencil-and-paper problem solving exercises, the solutions they offer do not always show the extent of visual processes which were involved (Moses, 1977).

In 1980 Moses conducted another research, this time examining gender and age differences in spatial visualisation, reasoning and mathematical problem solving tasks and how visual thinking exercises affected the differences. She divided the subjects, who were middle-class students in fifth grade, ninth grade and university level, into experimental and control groups and gave both groups seven pencil-and-paper tests as pre-test and post-test. Four of the tests administered were tests of spatial abilities: Mental Rotation, Punched Holes, Form Board and Hidden Figures. Two were reasoning tests: Nonsense Syllogisms and Reasoning, whereas the third was a problem-solving test comprising of ten non-routine mathematical problems. Correlations between scores on the problem-solving inventory and spatial abilities tests, reasoning and the 'degree of visuality' were all significantly different than zero. The author's conclusion was that instruction in visual thinking affected spatial and reasoning abilities, but this did not involve problem-solving achievement or the 'degree of visuality' (Moses, 1980).

Criticism of Moses' studies extend to her 'degree of visuality' construct, i.e. the method she used to obtain the scores on it but also to the items contained in the problem-solving inventory. Moses measured individual 'degree of visuality' by analysing the students' written solutions to the problems given, noting the number of occasions on which the participant made a graph, list or diagram, all of which she considered to be displays of spatial skills. In some of the problems it was required of the participant to draw a graph and such occasions could not have been treated as spontaneous 'display of spatial skills'. Also, there is a possibility that certain participants did not put the visual imagery on paper. The second point in criticism of her work is that the problem-solving inventories were of high difficulty. Mean scores in her second study were 1.22 out of maximum 10 for fifth-grade, 2.25 out of 10 for ninth and 3.33 out of 10 for university level (Lean & Clements, 1981).

4.2 SMPY (Study of Mathematically Precocious Youth)

One of the projects highly relevant for this topic is certainly SMPY (Study of Mathematically Precocious Youth). It is a longitudinal study designed to discover best methods for identifying and nurturing talent for STEM as Super and Bachrach suggested in their work (1957). Shea, Lubinski and Benbow (2001) followed 563 talent search participants who were identified in 1970ies via SAT (Scholastic Assessment Test) until the age of 13 as intellectually talented, among 0.5% within their peers of the same age. Among other things they were also assessed on spatial abilities. Biographical, educational and occupational criteria were collected 5, 10 and 20 years after identification. Those young adolescents who displayed higher levels of spatial ability at the age of 13 and who subsequently found math and science to be their favourite subject in high school, earned graduate and undergraduate degrees in STEM and made careers in this field as well. What is more, function analyses conducted at the three time points showed that spatial ability added incremental validity beyond the SAT-Mathematical and SAT-Verbal in predicting math-science criteria. Webb, Lubinski and Benbow (2007) used a less selective sample of 1060 adolescents who were identified in 1990ies, who were among the top 3% in ability, and showed that spatial ability has incremental validity over both SAT scales and various educational-occupational preference questionnaires over a five-year interval for predicting favourite high school courses, leisure activities which are relevant to STEM, focus of study at the university and in the end, the choice of occupation. The period spanned again from the initial identification at the age of 13 and monitoring continued until after high school. On the whole, spatial ability accounted for additional 3% of the variance in prediction of these criteria beyond the SAT and educational-occupational questionnaires. Apart from the adolescents whose outcomes ended up in the human sciences and other fields, those with STEM outcomes showed higher levels of spatial abilities at the age of 13 (Wai, Lubinski & Benbow, 2009).

Lohman, however, pointed one shortcoming of this research and the conclusions drawn from it. The authors did not use random samples of general population or even random samples of students with higher abilities. All participants were highly talented individuals, motivated to attend complex academic programmes for talented youth (Wai, Lubinski & Benbow, 2009).

4.3 Project TALENT research by Wai, Lubinski and Benbow

Wai, Lubinski and Benbow used data which Project TALENT yielded in order to examine the extent to which the spatial abilities assessed in adolescence is a significant characteristic among those participants who decide on pursuing careers in the field of STEM. Project TALENT's initial data collection in 1960 consisted of a random sample of high school pupils in the US. Pupils through 9th to 12th grade were assessed on a variety of tests and questionnaires during a one-week period. The entire sample consisted of 50 000 males and 50 000 females per grade, with a total of cca. 400 000. In the tests were included many measures designed to assess cognitive abilities (such as mathematical, verbal and spatial ability), general information tests (about content areas including art, biology, engineering, journalism and physics), measures of attitudes, interests and personality traits, as well as additional questionnaires consisting of questions relating to their lives (family, hobbies, school, health etc.). The Project also includes longitudinal data taken one, 5 and 11 years after graduation from high school (Wise et al., 1979) Wai, Lubinski and Benbow then examined the data and focused on those participants who reported being awarded academic titles and their choice of occupation (Wai, Lubinski & Benbow, 2009).

The authors used simplified form of the hierarchical model of cognitive abilities outlined by Carroll (1993), the radix scaling of cognitive abilities (Snow, Corno & Jackson, 1996; Snow & Lohman, 1989). This model groups cognitive abilities around three areas: mathematical, spatial and verbal, whereas G is a construct of general intelligence (Wai, Lubinski & Benbow, 2009).

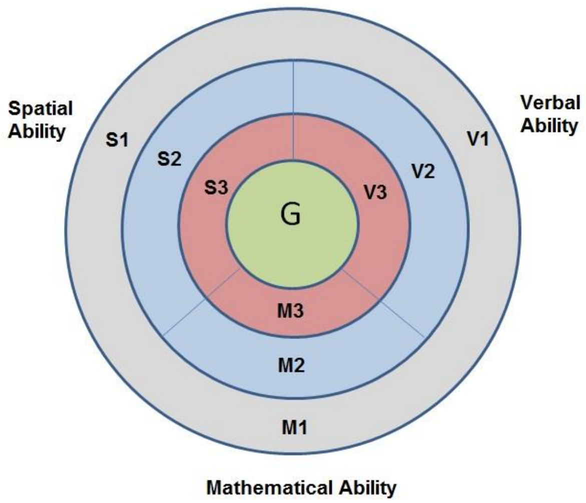

Figure 4. The radex of cognitive abilities (Snow & Lohman, 1989; Snow, Corno & Jackson, 1996).

The authors further formed three ability composites in order to measure the components found in the radix (Snow & Lohman, 1989; Wise et al., 1979). Mathematics composite consisted of four tests:

1. Mathematics information – consisting of 23 items examining knowledge of math definitions and notation
2. Arithmetic reasoning – consisting of 16 items examining reasoning ability needed to solve basic arithmetic problems
3. Introductory mathematics – consisting of 24 items examining math knowledge taught through the 9th grade
4. Advanced Mathematics – consisting of 14 items examining knowledge in algebra, plane and solid geometry, probability, logic, logarithms and basic calculus

Verbal composite consisted of following tests:

1. Vocabulary – consisting of 30 items measuring general vocabulary knowledge
2. English composite – consisting of 113 items measuring knowledge of English grammar, capitalisation, punctuation etc.
3. Reading composite – consisting of 48 items measuring understanding of given text

And finally, the spatial composite consisted of four tests:

1. 3D spatial visualisation – consisting of 16 items measuring the ability to visualise 2D figures after they have been folded into 3D objects. Each problem in this test consists of a piece drawn unfolded on the left side and five possible configurations of how the unfolded piece would look like when folded. It is required of the subjects to choose the correct configuration.
2. 2D spatial visualisation – consisting of 24 items measuring the ability to visualise 2D figures after being rotated otherwise undergone a transformation on a plane. Each problem in this test consists of a configuration on the left side and then five possible outcomes of the original configuration's rotation.
3. Mechanical reasoning – consisting of 20 items measuring the ability to understand relationships between gears, pulleys and springs, as well as knowledge to understand effects of basic physical forces. In this test problems consist of diagrams and pictures showing pulley, gear and spring configurations in which it is e.g. required of subjects to conclude in which direction a specific gear will be turning.

4. Abstract reasoning – consisting of 15 items as a nonverbal measurement of finding logical relationships in complex figure patterns. Each problem in this test consists of a set of figures which all share a common pattern. The subjects need to understand the pattern and then draw the figure which is missing (Newcombe, 2010).

The authors' hypotheses were:

1. Pattern discovered during SMPY on the three specific abilities will be mirrored by those in Project TALENT when compared against pertinent educational and occupational criterion groups.
2. Significance of spatial ability will increase as a function of more demanding STEM criteria (e.g. academic degree)
3. Conclusions from the Graduate Record Exam will mirror those of Project TALENT and SMPY.
4. Significant percent of adolescents with talent for STEM or other fields for which spatial abilities are required is not visible by the current selection procedures for the choice of career in STEM.

Results showed that 45% of those holding STEM PhDs, 30% of those holding STEM master's degrees and 25% of those holding STEM bachelor's degrees were within the top 4% on spatial abilities eleven years earlier. The authors concluded that the importance of spatial ability for STEM fields increases along with the academic degree the subjects were awarded later. It is interesting to note, with regard to the question of whether spatially gifted students were missed by current talent-searching mechanisms which focus only on mathematical and verbal talent, that 70% of the top 1% in spatial ability did not make it into the top 1% on either mathematics or verbal composite and yet these individuals are highly gifted in spatial ability and yet they earned STEM and visual arts degrees (Wai, Lubinski & Benbow, 2009).

4.4 Sorby et al research on spatial skills as predictors of success in first-year engineering

S. Sorby, E. Nevin, A. Behan, E. Mageean and S. Sheridan conducted a research on the spatial skills as predictors of success in first-year engineering during the first semester in 2013 at the

Dublin Institute for Technology among students studying civil engineering, mechanical engineering and common engineering, which results were then compared with those of students of architecture, architectural technology and computer science. The authors administered the PSVT:R and MCT tests during regular class times and in some cases both tests were administered on the same day, although in most of the cases the students solved only one of the two tests. At the end of the semester, students' grades in their regular courses were collected on a numerical scale of 0 to 100. It should also be noted that in Ireland a minimum of 40% score is required for a passing grade in the course (Sorby et al, 2014).

Students in Ireland take an exam in their final secondary school year which is called Leaving Certificate (LC). Points are collected from a student's six best subjects with a maximum of 600 points (a maximum of 100 for each subject). Higher education in Ireland also distinguishes, among other levels, Level 7 which is called Ordinary Bachelor's Degree and Level 8, the Honours Bachelor's Degree. For entry e.g. on the Level 8 engineering programme it is required of students to have been awarded a minimum grade of C (55%) in the higher level secondary school mathematics exam, whereas to enrol into the Level 7 engineering programme a minimum grade D is required (40%) on the lower level mathematics exam. In other words, it is logical from these requirements that students on Level 8 will have a higher mathematical pre-knowledge than those on Level 7 (Sorby et al, 2014).

There were more than 800 students who completed one or both of the tests, whereas for those who completed both correlation between the two tests was calculated (r =0 .634, p < 0.001), which findings indicate that an individual score on one test is a good predictor of the score on the other test. The authors focused on gender differences on the tests but also on the differences between students studying different disciplines. The average for the PSVT:D was 20.46 out of maximum 30 (68.2%) and 11.5 out of possible 25 (46.2%) for the MCT. The results obtained indicate that the students found the MCT test more difficult than the PSVT:R. With regard to the gender differences, on the PSVT:R men averagely scored 20.51 and women 16.54 out of maximum 30, whereas on the MCT men averagely scored 10.24 and women 8.53 out of maximum 25. The authors compared average scores with regard to disciplines and also with similar programmes, comparing them against each other but also against results obtained using the PSVT:R and MCT at other institutions, such as in the US, Poland and Germany (Sorby et al, 2014).

When compared against other results, spatial skills of the Irish students are behind those of students in the US and other European countries. One possible explanation the authors offered was the age – while Irish students are typically 17-19 years old when they enrol a university, the students in the US and Poland are 18-19 and in Germany 19-20. Second possible explanation was that in Ireland a greater percentage of secondary school pupils continue with university education, and so it would be logical that the scores of the Irish students on the spatial tests when compared against the small percentage of the elite which enrols at universities in for instance Germany. With regard to the students of computer science, their scores on PSVT:R are among the highest and their scores on MCT are among the lowest. The authors offer the explanation that computer science students often play computer games, which have been shown to enhance spatial skills (Green & Bavelier, 2003, p.534.) and given that they involve rotating objects or people in space but do not involve determining cross-sections of objects. Low scores of Architecture students the authors explain by the relatively large percentage of women in that programme (30%) when compared against percentage of female students in other programmes (less than 10%). Generally spatial skills of students studying in the ordinary programmes are lower than those in the honours programmes, which is expected since it has been shown that there is a connection between spatial skills and mathematical abilities (Fennema & Sherman, 1977) and of course, students in the honours programmes have higher mathematics entrance scores (Sorby et al, 2014).

From the results obtained after analysing the students' scores in their regular courses the authors concluded that LC Points are a good predictor of success in higher level mathematics in the honours programme ($p < 0.0001$) but not for those in the ordinary programmes. One explanation offered by the authors is that the classes at the university are very similar to the classes in high schools and thus result in similar patterns for these groups. No significant correlations have been found between performance on the MCT and at the introductory mathematics course. The authors also wanted to examine whether low spatial skills reflect in low performance on the mathematics course. To this end they fixed the score of 11 out of maximum 25 on the MCT test as threshold score between weak and good spatial abilities, which idea stems from the data gathered from previous studies in the US where variable categorisation was required into weak, average and high, based on their score on the MCT (Hamlin, Boersma & Sorby, 2006). The variable of success on the mathematics course was categorised as receiving a passing grade or more (≥40%), with failure categorised as failing the course or not completing it at all (<40%) (Sorby et al, 2014).

For the honours students the LC points seem to predict success on the mathematics introductory course and not spatial abilities, as significant correlations suggest. Weak visualisers had success rate of 60.5%, average score on the mathematics course 49.4% with average LC score of 395.8 ($p < 0.05$), whereas good visualisers had success rate of 60.5%, average score on the mathematics course 46.8% with average LC score of 420.5. However for the students in the ordinary mechanical engineering problem the results suggest the opposite situation. Weak visualisers had success rate of 65.5%, average score on the mathematics course 49.9% with average LC score of 330.2, whereas good visualisers had success rate of 93.3%, average score on the mathematics course 62.9% with average LC score of 347.5. Those students who did not succeed on the mathematical course were also in the weak visualisers group. However, for the students in Level 7 Civil Engineering course both LC points as well as spatial skills could be predictors of success on the mathematics course. In the weak visualisers group success rate was 52.2% ($p<0.1$), average score on the mathematics exam 38.36% ($p<0.05$) and average LC score of 280.3 ($p<0.05$), whereas in the good visualisers group success rate was 60.5%, average math score 63.33% and average LC score 365.0. However, given that only three students were from the good visualisers group the authors could not offer any conclusions (Sorby et al, 2014).

The authors have concluded that students with better mathematics pre-knowledge are also more successful in their introductory mathematics course and as such, it can be concluded that high prior performance is an indication of success in mathematics course at university level. For students with low pre-knowledge in mathematics no correlation has been between prior knowledge and performance. It has been found in previous studies in the US that spatial skills act as predictors for grades and persistence in engineering programmes. However this was one of the factors which was not examined in the previously covered research (Sorby et al, 2014).

4.5 Development of a course to improve spatial abilities of engineering students by Martín - Dorta, Saorín and Contero

The developed course was offered to the students alongside of other remedial courses offered to first-year students in order to improve their knowledge in basic subjects and participation in such courses is voluntary. Participants consisted of 40 students (25 males and 15 females) in civil engineering programme at the University of La Laguna, Spain. Most students were 18-20 years old. The authors used freeware version of Google SketchUp 5, which allows users to build and modify 3D models quickly and easily. Apart from this software, the authors used a set of 24 objects made of aluminium and of approximate dimensions 60x55x45mm, which the students used for orthographic view definitions, whereas they used cuts and sections for geometrical definitions (Martín-Dorta, Saorín, & Contero, 2008). Working schedule is outlined in the following table:

Level	Week	Description
Level 1: Initiation	1	• Building 3D models based on the appearance of the aluminium objects • Creating a daily object
Level 2: Improvement	2	• Creating 3D models corresponding to parts given by their axonometric projection
Level 3: From orthographic views to 3D models	3	• Constructing 3D models of parts which are represented by their orthographic views • Group work

Table 2. Martín-Dorta, Saorín and Contero remedial course working schedule, (Martín-Dorta, Saorín, & Contero, 2008)

In the first part of the course, the students were getting familiar with Google SketchUp 5. After they acquired basic skills for work in the software, they were required to choose one of the aluminium objects and to create a corresponding 3D model of it. They were recommended to first sketch the object on paper and then sketch it in software. As homework they were given the task of creating a daily object, measuring it and then sketching it in SketchUp 5. In the second part of the course it was required of students to create 3D models corresponding to the parts given by their axonometric projection (Pérez and Serrano, 1998). The authors then assessed students' ability to interpret the 3D model from a 2D perspective. This phase of work did not include any instructions on orthographic views. In the third part of the course it was required of the students to construct a 3D model of parts represented by their orthographic views. They were given one-hour instructions on orthographic views and afterwards discussed proposed group exercises. The authors state that the students found this phase of work the most complex, since they had to visualise and construct and mental image of the object based only on its orthographic projection. The authors proposed formation of working groups, not only to solve the problem given to them but also to promote a climate of group work and cooperation (Martín-Dorta, Saorín, & Contero, 2008).

The authors administered the MRT and DAT:SR tests before and after the course, calculating also the difference between the results. Gain differences on the MRT test was 5.48, whereas on the DAT:SR was 8.30. Differences in improvement on the MRT for men was 5.96 whereas for women was 4.67, whereas differences in improvement on DAT:SR for men was 8.16 and for women 8.53. The authors further concluded from the obtained results that the course employing Google SketchUp had a measurable and positive impact on the spatial abilities of the participants, with improvement on the MRT averagely 5 points and on the DAT:SR 8 points, regardless of gender (applying *t*-test for independent series $p = 0.552$ and $p = 0.868$, respectively) (Martín-Dorta, Saorín, & Contero, 2008).

With regard to previous similar research, the authors held three remedial courses for students with problems with spatial abilities during the academic year 2004/5. First course was based on the paper-and-pencil exercises, second one employed a web-based application featuring exercises for spatial abilities development whereas the third one employed a sketch-based modelling application e-CIGRO, developed by the REGEO Research Group (Regeo Research Group, 2008). The subjects were students who scored among the lowest 20% on pre-test. In order to be able to compare results with the pervious researches which have just been outlined, the authors narrowed down the sample on just the percentage of students who scored among the lowest 20%. ANOVA analysis was performed and with regard to the MRT test, the effect of the type of course was not significant ($F_{3,60} = 0.83$ and *p*-value $= 0.483$), whereas with regard to the DAT:SR the course type was significant ($F_{3,60} = 4.35$ and *p*-value $= 0.008$). Furthermore, Tukey test indicated that the mean improvement in group 2 (working with the web-based application) was significantly lower when compared to the group 1 (paper and pencil) and group 4 (Google SketchUp). Group 3 (sketch-based modelling) showed no differences (Martín-Dorta, Saorín, & Contero, 2008).

4.6 Measuring spatial ability of students of mathematics education in Croatia by Šipuš and Čižmešija

The aim of the authors was to analyse spatial skills of future mathematics teachers studying at the Department for Mathematics, University of Zagreb, Croatia using the MCT. The testing was performed at the beginning and at the end of the 1st semester in academic year 2009/10. The sample consisted of 98 students (29 male and 69 female) in their first year of Bachelor Mathematics Education Programme (ME1) and 32 second year students (6 male and 26 female) in Master Mathematics Education programme (ME5). In order to avoid the effect of learning the test, the authors rearranged the items in the test. It was also one of the authors' aims to compare the results obtained from ME1 and ME5 students with those studying at different STEM universities (Šipuš & Čižmešija, 2012). The sample consisted of:

- 116 students (50 male and 66 female) studying at the Faculty of Civil Engineering (Eng1),
- 102 at the Faculty of Architecture (Eng2),
- 204 at the Faculty of Geodesy (Eng3),
- 192 Faculty of Mining, Geology and Petroleum Engineering (Eng4).

The authors reported the following average results per different groups: ME1 scored averagely 9.97, ME2 14.66, Eng1 15.83, Eng2 13.06, Eng3 11.87 and Eng4 8.71 out of maximum 25 points. That authors observed that ME5, Eng1, Eng2 and Eng3 students achieved better results than ME1 and Eng4 students, what they confirmed employing a t-test which showed statistically significant differences between the results of Me1 and Me5 students ($p=0.01$). They also showed statistically significant differences between the results of ME1 and Eng1, as well as Eng2 and Eng3 students ($p=0.01$). One possible explanation the authors offered for the differences in spatial abilities of ME1 and ME5 students is that ME5 students are less in number and also that they had opportunities to improve their spatial abilities through various courses they covered while studying. Lastly gender overall differences in performance on the MCT test were examined. Average scores for men was 12.3 and 9.10 for women in the ME1 group, 19.00 for men and 13.65 for women in the ME5 group and 17.08 for men and 14.8 for women in the Eng1 group. It has been shown that male students perform statistically significantly better than female students (p=0.01). The authors emphasise that female students studying Mathematics Education outnumber male students, with 70% women in ME1 and 81% of women in ME5. Comparison of ME1 and Me5 female students' performance on the MCT with the Eng1 female students' performance shows that women in the Eng1 group scored statistically significantly better (p=0.01) (Šipuš & Čižmešija, 2012).

4.7 Research on the correlation between primary mathematics teachers' gender, academic success and spatial ability by Turgut and Yilmaz

The authors conducted a correlational study which had for its aim to determine relationships among future primary school mathematics teachers' gender, academic success and spatial abilities. The sample consisted of 193 subjects (111 female and 82 male). Instruments used for measurement was the spatial ability test developed by Ekstrom et al (1976) consisting of the Spatial Orientation Ability Test (SOAT), which consists of two subtests, Card Rotation Test (CRT) and Cube Comparison Test (CCT), and Spatial Visualisation Test (SVAT), also consisting of two sub-tests, namely Paper Folding Test (PFT) and Surface Development Test (SDT) . Total average scores on the SOAT amounted to 176.80 out of maximum 202 and on the SVAT 38.92 out of maximum 80. Total average score on both tests for all participants was 213.48 out of maximum 282. With regard to gender, the performance on the SOAT was in favour of women, who averagely scored 175.52, whereas men averagely scored 174.09. The performance on the SVAT was once again in favour of women, who averagely scored 39.50, whereas men averagely scored 38.35 (Turgut & Yilmaz, 2012).

In order to examine the correlation between academic success of future primary school math teachers and spatial ability the researchers used Pearson Product Moment Correlation Coefficient. Statistically significant correlation has been found (r=0.36, p< 0.01). From the results the researchers also concluded that there was also a statistically significant correlation between spatial orientation ability and academic success (r=0.29, p<0.01), as well as between spatial visualisation ability and academic success (r=0.32, p< 0.01). With regard to the difference between gender and spatial ability, the distribution of the spatial ability scores needed to be examined, for what the researchers used the Kolmogorov-Smirnov normality test. It has been concluded that distribution of the scores with regard to gender does not have characterisation of normality ($p<0.05$). Further application of the non-parametric Mann-Whitney U test indicated that there is no statistically significant difference between the scores on spatial ability and gender ($U=4297.5$, $p> 0.05$). Researchers lastly used Pearson Product Moment Correlation coefficient to determine the relationship between spatial orientation and spatial visualisation abilities and it has been observed that spatial orientation and spatial visualisation of future primary school mathematics teachers are positively correlated ($r=0.42$; $p<0.01$) (Turgut & Yilmaz, 2012).

4.8 Pre-graphics course by Sorby and Baartmans

Sorby and Baartmans (2000) developed a pre-graphics course in 1993 on the Michigan Technological University which had for its aim to improve spatial skills of freshmen. Tools used for assessment of the development of these skills were the Purdue Spatial Visualisation Test: Rotations, The Mental Rotation Test, The Mental Cutting Test and The Differential Aptitude Test: Space Relations. The Purdue Spatial Visualisation Test tests a person's ability at the second stage of spatial development, the Mental Rotation Test assesses a person’s ability to visualise rotated solids, in the Mental Cutting Test the participants are asked to choose the correct cross-section which results from cutting a criterion figure with an assumed plane, whereas The Differential Aptitude Test requires of a participant to choose the correct 3-dimensional object from four alternatives which would result from folding the given 2-dimensional pattern (Sorby & Baartmans, 2000).

The goal of the authors was among other things to help female colleagues studying at MTU to enhance their 3D visualisation skills. In 1985 Baartmans conducted a research study at the MTU. The sample consisted of 365 first year students (65 women and 300 men) who took Mechanical Engineering as the focus of their studies. Before the beginning of the course pretest in the shape of PSVT:R has been distributed to the subjects. Multiple regression analysis has shown that the most significant predictor of success in the freshman graphics course was the students’ score on this test. Two other factors which were found to be significant in predicting students' success were mathematics subtest score and a combination of prior experience in shop, drafting and solid geometry. Scores on the spatial visualisation test were 20.9 for women and 24.2 out of 30 for men. Their scores significantly improved after the course, but women still remained behind men in their achievements on the test: 23.3 for women and 25.6 for men (Sorby & Baartmans, 2000).

In 1993 the authors wrote a textbook to be used in their new introductory 3D spatial visualisation skills course called *GN102, Introduction to Spatial Visualization,* consisting of following topics:

Week 1 – course introduction	Students were introduced to the need for visualisation skills in fields of engineering, medicine, architecture, chemistry and mathematics.
Week 2 – isometric and orthographic sketching	Students were given a set of cubes and were told to construct a building according to coded plans. They learned how to make isometric and orthographic drawings of the building using grid paper.
Week 3 – orthographic drawings and applications	Objects which contained inclined surfaces have been demonstrated and orthographic and isometric sketches of these objects have been made of these objects.
Week 4 – pattern development	Flat patterns which are to be folded into 3D solids are studied.
Week 5 – Two and three coordinate drawing	Students were shown how to locate specific points in space and they used a table of coordinate data in order to draw wire-frame models. Surveying application using traverse data was introduced.
Week 6 – Translation and scaling	Transformations in 3D space were introduced and students were required to draw objects after translation and scaling.
Week 7 – Object rotation	Students were required to work with objects made of snap cubes and sketch isometric views of the objects after rotated about one or the more axes.
Week 8 – reflection of objects and applications	In order to construct reflected images of objects students used Miras in class.
Week 9 – Cross-sections of solids	Students are taught how to graphs planes in 3D space and they examined cross-sections for cubes, cones and cylinders of different orientations.
Week 10 – surfaces and solids of revolution and the intersection of solids	Students were required to sketch the surface or solid created by the rotation of a planar figure around an axis. The also sketched the shape of the planar figure which was rotated given the surface/solid of rotation.

Table 3. Introduction to Spatial Visualization working schedule, Sorby and Baartmans study (Sorby & Baartmans, 2000)

In 1993 students who enrolled in mechanical, civil, environmental, geological and general engineering were given the PSVT:R and a background questionnaire as part of their freshman orientation lecture. Sample consisted of 535 students (418 male and 117 female). Average percent of correct answers on the test was 79.6% for males and 68.1% for females. From further statistical analyses of the background questionnaire and the PSVT:R eleven factors have been studied, four of which have shown to be significant predictors of success on the PSVT:R test. Those were: 1) play as children with construction toys, 2) gender, 3) math scores, 4) previous experience in design-related courses like drafting, mechanical drawing etc. Factors which were not significant for the PSVT:R score were: 1) age, 2) right/left handedness, 3) previous experience in high school geometry courses, 4) participation in industrial arts courses in high school, 5) playing video games, 6) previous work experience which involved spatial skills, 7) participation in sports which involve placing an object in a specific location (e.g. baseball, hockey etc.). Compared to the mean test scores which was 15.5%, on the post-test the students scored a mean test score of 24.7%. The course has been taught in subsequent years to the incoming first year students and different tests for pre- and post-testing have been administered, such as MRT, MCT and DAT:SR (Sorby & Baartmans, 2000).

4.9 Spatial ability, visual imagery and mathematical performance study by Lean and Clements

Lean and Clements conducted a study in 1981 which had for one of its aims to answer the question of whether persons who prefer to use visual imagery when dealing with mathematical information are likely to perform better on certain mathematical tasks than people who prefer verbal-logical mode. Their subjects were 116 first year Engineering students of the University of Technology, in Papua New Guinea. Mean age was 19.6 years, whereas there were 114 males and 2 females. A battery of five spatial tests was administered to the participants during the first two weeks of their course: 1) spatial Test EG by I. MacFarlane Smith, 2) Spatial Test II by A.F.Watts, D.A. Pidgeon and M.K.B. Richards, 3) Gestalt Completion Test by R.F. Street (1931), 4) Standard Progressive Matrices, Set D, by J.C. Raven (1938), 5) 3D Drawing Test by M.C. Mitchelmore (1974). During a further session in the third week they were given a mathematics test and a questionnaire developed by Suwarsono. During their course the subjects were additionally given two more mathematics tests: 1) ‘pure’ mathematics test consisting of 24 items dealing with routine mathematical knowledge, 2) ‘applied’ mathematics test consisting of 27 items dealing with understanding of physical and mechanical concepts. Suwarsono’s questionnaire, which was developed in 1979 in Melbourne, consists of two parts: 1) 30 mathematical word problems, 2) written descriptions of different methods normally used by students solving problems in first part, normally ranging from three to five possible methods outlined. Students are then asked to try to solve problems in part one and then indicate which of the outlined methods in part two they used and, if the method they used was not outlined, to give a detailed description of it (Lean & Clements, 1981).

Multiple regression analysis suggested that spatial ability did not have a large influence on the mathematical performance on tests. Moreover, the results showed that students who preferred to process mathematical content using verbal-logical mode outperformed the more visual students on both mathematical and spatial tests. This result seems to be in contradiction with results of similar studies and the authors offer one possible explanation for this – straightforward, routine problems in the ‘pure’ and ‘applied’ mathematics tests, whereas in other studies more complex, non-routine problems were used and also to the possible developed skill to abstract readily and thus avoid use of visual imagery. As one of the faults of the study the authors outline the fact they did not measure many of the non-mathematical variables such as student motivation, working habits or language competence (Lean & Clements, 1981).

4.10 Study in training generalised spatial skills by Wright, Thompson, Ganis, Newcombe and Kosslyn

R. Wright, W. L. Thompson, G. Ganis, N. S. Newcombe and S.M. Kosslyn (2008) conducted a study which had for its aim to examine whether intensive long-term practice leads to changes which go beyond specific tasks and stimuli. The participants were 31 students (14 male, 17 female), most of them undergraduates with an average age of 23.4 recruited via Harvard University Psychology Department website, who voluntarily took part in practice sessions. The study consisted of three tasks, whereas problems presented consisted of reference image on the left side with comparison image on the right side against black background. For each task the authors included a comparable amount of easy, medium and hard problems (Wright et al, 2008).

1. First task was MRT, a computerised version of the MRT, measuring the ability to compare a pair of 3D objects showed in different orientations and where participants were required to decide whether they were identical or mirror images, stretching over 48 basic block configurations which led to 288 unique items. 9 female and 8 male participants were trained on this test.
2. This task consisted of MPFT, a computerised adaption of the task designed by Shepard and Feng (1972). However in addition to the original 2D unfolded cubes, the authors included a reference 3D cube image with the aim of creating a comparison task analogous to the other tasks they used in their study. Whereas originally Shepard and Feng created 165 unique items, the authors stretched the number of items to 255, across different difficulty levels. Easy level referred to whether a single square had to be folded in order to reach solution, two or three squares referred to medium level and 4-7 squares to hard level. 8 female and 6 male participants were trained on this test.
3. The last task was based on the one developed by Morrison et al (2004) in which it is required of the participants to compare relationships between two words in each pair of words presented simultaneously and to decide whether the relationship between the words in the left pair is the same as the one between the words on the right side. Authors again grouped the individual problems into easy, medium and hard level, according to mean RTs for correct solutions, as gathered from previous research. However, only a subset of the items was used due to the difficulty categorisation (Wright et al, 2008).

The participants first completed 12 trials in order to get familiar with the system, with of course different items than those given in the experimental trials. They gave their responses by pressing keys labelled ‘same’ and ‘ different’ and it was required of them to do this as quickly and as accurately as possible (up to 6 seconds). After completing the initial laboratory session the participants were divided into two groups: those practising MRT and those practising MPFT. This phase of research was conducted via internet. The participants were required to conduct daily sessions lasting about 15-20 minutes throughout 21 days in a row. The practice phase consisted of 114 trials. The items presented in sessions were presented randomly and thus participants could miss up to three sessions without needing to be cut from the study. One day after completing the practice phase, the participants were retested again on all three tasks in the laboratory. This procedure was identical to the one of initial laboratory session, with the exception that about half of the items presented in this phase were encountered once, during the initial laboratory session, whereas half was completely new (Wright et al, 2008).

Authors' findings show that practice symmetrically transferred to spatial tasks. Moreover, it has been found that improvement is greater in the non-practiced spatial task than in the VAT. Speed with which the participants acquire skills and transfer within tasks could have been affected by differences in the frequency and spacing of sessions. In Wright et al research transfer effects were observable after 7 hours of practice spanning over three weeks. Terlecki et al (2008) found that transfer occurred after 12-14 hours of practice spanning over 14 weeks. Post-testing showed transfer of practice gains to newly introduced items for the task which has been practised, but also transfer to other spatial tasks which have not been practised. Moreover, improvement in the spatial task which has not been practised was greater than that in the VAT, from what the authors concluded that the observed improvement was not because of easier computerised testing (Wright et al, 2008).

4.11 Developing spatial visualisation with 3D modelling by Šafhalter, Glodež and Bakračevič Vukman

The research sample consisted of 22 students of the 9th class (age 14-15 years), with 10 male and 12 female pupils. 14 pupils have been sorted into the experimental and 8 into the control group. In December 2010 a pre-test has been administered to the subjects, which consisted of a modified spatial tests battery and a questionnaire. Spatial tests battery consisted of PSVT:R, MRT and DAT:SR, whereas the questionnaire measured the pupils' way of learning, thinking and communicating. The modified spatial tests were administered once again in May 2011 (Šafhalter et al, 2012).

Subjects from the experimental group participated in 3D modelling in Google SketchUp. Their work consisted of modelling objects based on their isometric and perpendicular projections, beginning with simple objects and gradually progressing to more complex objects (Šafhalter et al, 2012).

Comparison between the results on the pre-test and the post-test showed that the experimental group achieved 2.65 points more than on the pre-test, whereas the control group achieved 1.0 points more. Analysis also showed that there were no statistical differences with regard to progress in spatial visualisation of the experimental group when compared with that of the control group. Results have also shown that there were no statistically significant differences between male and female participants with regard to the progress of spatial visualisation. The questionnaire which measured the pupils' way of thinking and learning helped to sort the pupils into groups according their way of thinking. Results showed that there were statistically significant differences between pupils who employed visual perception style when compared to others (Šafhalter et al, 2012).

4.12 Gender differences with regard to spatial abilities

Although gender differences will not be a topic of research in this work, the sheer volume of research focusing on them requires their mentioning. Eals and Silverman write that gender differences with regard to spatial abilities are evident in favour of males "universally across regions, classes, ethnic groups, ages and virtually every other conceivable demographic variable" (Eals & Silverman, 1994, p. 95). Research has shown that superiority of males is mostly recognisable in tasks of mental rotation, with less evident differences in orientation and no differences in visualisation (Harris, 1978; Linn & Petersen, 1986). Studies also emphasise the effect of hormones on spatial abilities, with oestrogen negatively affecting spatial abilities whereas testosterone has been found have a non-linear effect on spatial abilities (Kimura, 1996; Moffat & Hampson, 1996). Apart from this purely chemical reason for gender differences with regard to spatial abilities development, it has been concluded that they are a result of biological factors (Bock & Vanderberg, 1968; McGee 1979a) i.e. that the level of spatial abilities is hereditary.

Environmental theories on the other hand suggest that cultural, social, gender-roles, stereotypes and educational factors play a significant role in differences between genders with regard to spatial abilities (Mann et al, 1990; Belz & Geary, 1984; Tracy, 1990; Harris, 1978). Atop of this, researchers representing environmental approach suggest that problem solving strategies and skills, mathematical background and even musical background are possible roots for spatial abilities development and can explain gender differences (Clements & Battista, 1992; Mislevy et al, 1990; Michaelides, 2002; Wheatley et al, 1994; Heitland, 2000; Robichaux & Guarino, 2000). One of the environmental explanations for gender differences can also be found in the choice of toys and play in the early childhood. Many stereotypically masculine activities such as construction blocks and model building enhance spatial abilities development (Caldera et al, 1989; Caplan & Caplan, 1994). Moreover, from infancy throughout childhood, boys and girls receive different messages about how suitable a particular toy for them is from their parents, caregivers and siblings (Caldera et al., 1989). Furthermore, Hoyenga and Hoyenga (1993) suggest socialisation plays an important role in the level of development of spatial abilities. The authors found a correlation between the level of spatial abilities development and the frequency of playing with toys which encourage the development of spatial abilities such as building blocks, mechanical contraptions which can be combined and put together. Such toys are more frequently given to boys to play with and boys are encouraged to do so, whereas girls who had a lot of experience with play with such toys exhibit significantly less inferiority in spatial abilities. Moreover, it has been found that among Eskimos no significant gender differences in spatial abilities exist (Neisser et al, 1996: according to Zarevski, 2000, p.116).

Hoyenga and Hoyenga (1998) conducted a meta-analysis of research of gender differences in spatial abilities and found a size effect of 0.43 in the research by Hyde (1981). Differences which have been identified were on the higher levels of total cognitive efficacy, where more complex factor structures are present. In later meta-analyses size effect has been calculated, with the biggest effect of 0.93 found in the mental rotation tasks, average of 0.64 in the tasks dealing with field independence (e.g. water level tasks) and 0.13 which has not been found as statistically significant in spatial visualisation tests (e.g. paper folding tasks and DAT:SR) (Zarevski, 2000; p.115).

In 1989 Jerončić, Eterović and Zarevski (1989) conducted a research on differences in spatial and verbal abilities as well as perceptive restructuring with regard to the gender function and gender roles among fourth-grade secondary school pupils. Whereas for verbal abilities no statistically significant differences have been found, for other two abilities the authors found differences in favour of male participants. Only for spatial abilities differences have been found also with regard to gender roles – masculine participants without regard to their biological gender score highest. This research is in line with the broad meta-analysis by Signorella and Jaminos (1986) (Zarevski, 2000; p.117).

Although the reasons for gender differences with regard to spatial abilities are far from conclusive, it is evident that the differences have an impact on science and mathematics education. Some studies have shown that gender differences become evidently smaller with additional training or in the end eliminated them completely (Feng et al, 2007). For instance, Terlecki et al (2008) showed that females who initially scored poorly on spatial abilities tests improved slowly during the course of the experimental programme, but with time improved more. On the other hand males and females who scored initially high showed early improvement in the course of the training. The authors argue that the learning curve is important because if learning period is not sufficiently long it will appear that females benefit less from the training and show less improvement than the male participants (Terlecki, 2008).

5 Software and spatial abilities

It has been shown by research that spatial skills training using methods such as pencil-and-paper and work with physical objects is highly valuable, however the software development boom, especially freeware software, has brought a myriad of possibilities. It means that software which can be used for spatial skills training is not only accessible to schools or those who pay for it, but can be accessed and used by anyone who owns a computer and has access to the internet. It is beyond the scope of this work to list all of the known software, freeware or otherwise, which has been used or has the potential to be used as a spatial abilities training tool, however it is hoped that it will provide an adequate image of the topic.

Let us shortly turn to the criticism of dynamic mathematical software use. Researchers argue that the use of this software cannot replace the rigorous analytical thought processes in mathematics. While it can help develop some abilities and help subjects to solve a mathematical problem by providing a clearer picture of it, Sinclair (2003) reports that students involved in her study involving “pre-constructed, web-based, dynamic geometry sketches in activities related to proof at the secondary school level” showed "diagram bias", i.e. they were too accustomed to being presented with diagrams which reflected the problem and which were not entirely accurate in the sense that were not drawn to scale, which is why they mistrusted the accuracy of the Cabri or Sketchpad diagrams in their dynamic geometry programs (Sinclair, 2003). One further point which needs to be addressed is that the fact that students learned how to use the software and have mastered it does not imply they have mastered the underlying mathematical concepts. As a side note it is interesting to observe that the use of mathematical software can change students' attitude toward mathematics altogether (Reed, Drijvers & Kirschner, 2010).

5.1 MathPad© by LaViola and Zeleznik

The authors pointed out that in explanation of mathematical concepts and solving of mathematical problems ‘static’ methods such as drawing diagrams and illustrations normally only help in the initial problem examination, but that are not very helpful when it comes to analysis, which is especially evident in problems which entail complex spatial relationships or even in simple problems entailing natural mappings. When such sketches are animated it comes to a discrepancy between what they see and their ideas about motion, which leads to guesses. The authors designed MathPad, a prototype application for mathematical sketches (LaViola & Zeleznik, 2007).

MathPad incorporates a modeless gestural interaction paradigm which enables users to create handwritten mathematical expressions and free-form diagrams. An interface of this type has been used for: cooperative object-oriented design (Damm et al, 2000), conceptual 2D design (Gross & Do, 1996), conceptual 3D design (Igarashi et al. 1999; Zeleznik et al. 1996), musical score creation (Forsberg et al. 1998) etc. However one advantage of Mathpad is that its interface is modeless, which allows fluid transitions from e.g. drawing to writing down mathematical expressions. Authors' criticism of other applications of similar type such as Mathematica, Maple and Mathlab is that mathematical notation is unconventional when compared to concepts which are intuitive in handwritten mathematics expressions. One of the authors' goals was to design an application which would support computational activities including formula manipulation, diagram rectification and animal, which is accomplished by parsing the user's input or by allowing the user to perform gestural operations (LaViola & Zeleznik, 2007).

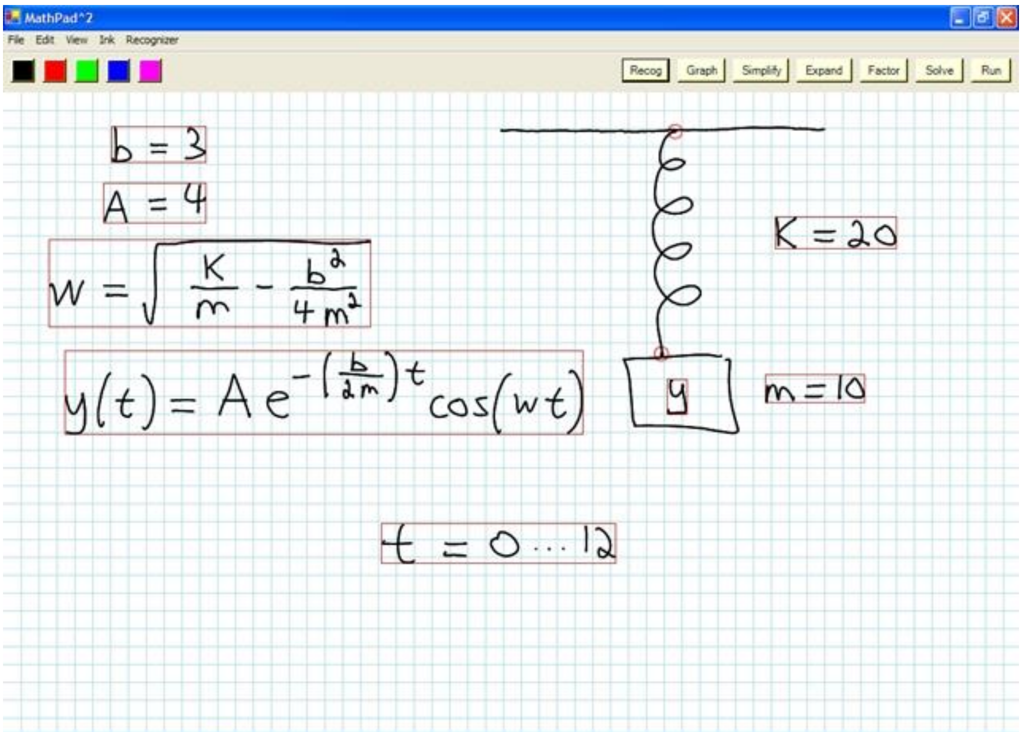

Figure 5.MathPad user interface.

Retrieved on the 15th of January 2017 from ***http://cs.brown.edu/~jjl/mathpad/***

Writing mathematical expressions and diagrams in MathPad is done by drawing with a stylus, whereas one complication would be how users can erase erroneous input. The authors designed a scribble erase gesture i.e. the user scribbles with the pen back and forth over the piece of input which he wishes to be deleted. The one obvious drawback of this feature, i.e. that the user may perform the gesture without meaning to delete anything, has been resolved by the authors by introducing an obligatory tap which follows the erasing gesture. One difficulty which arose with parsing the mathematical expressions was that it was hard to design the application in such a manner that it would algorithmically distinguish between mathematical lines, when some of the input is closely spaced or in an unusual arrangement. The authors therefore designed a manual segmentation of the input, i.e. the user needs to select a single mathematical expression by drawing a lasso, followed by a tap as by the erase gesture. The authors also included feedback into the MathPad application – the parsed mathematical expression with canonical versions of the original strokes. The reasons for this were 1) users can often distinguish characters and symbols in their own handwriting and wish to preserve the distinguishing characteristics of their handwriting style – the look, feel, spatial relationships of their notations because of aesthetics, subtle information detail and easy editing (Zanibbi et al. 2001). In case the there was an erroneous detail in the recognised expression, the users can simply scribble erase the details in question and rewrite them (LaViola & Zeleznik, 2007).

The process of drawing diagrams in MathPad is the same as with mathematical expressions, with one exception that they do not require recognising. MathPad uses nailing diagram components, which uses a concept of 'nails' in order to pin a diagram element to the background or a point on an element of the diagram to another element of the diagram. Moving diagram elements results in either the other element moving to stay attached at the nail if it has only one or stretches in such a fashion that all its nails keep their positions of attachment. In the user interface this technique looks as follows: the user creates a nail by encircling the appropriate locating on the diagram and tapping inside of it. The application links all elements which intersect the circle and can be recognised as a small red circle where the nail's location is. In turn, grouping diagram components is done if the lasso is drawn around the diagram strokes, followed by a tap. The application employs the Microsoft Tablet

PC SDK Divider API to distinguish between the lassoed strokes as being text or drawing (LaViola & Zeleznik, 2007).

With regard to the associations between mathematical expressions and drawings, MathPad allows them to be made both explicitly and implicitly. To create an implicit association, the users are required to draw a variable name or value close to the drawing and then use the mathematical recognition gesture. For explicit associations the user needs to draw a line through the mathematical expressions and tap on the drawing. In both methods the user visualises which drawings, labels and expressions he wants to associate. MathPad's ToolSet offers a range of computational functions such as graphing, solving, simplifying and factoring. Graphing is executed with a simple line gesture which starts at the function and ends at the intended graph location. The application recognises this gesture because it is too long to be a mathematical symbol and has no intersections or cusps. Process of equation solving is initiated when the user make a squiggle gesture which begins inside the box of recognised mathematical expression, and closely related gestures are employed for simplifying and factoring expressions. With regard to associating drawings with expressions, the drawing should be associated only if that particular expression participates in the computation of any variables falling in the category of the label family of the drawing's variable label. In other words, if the drawing is labelled with a certain label, MathPad searches for expressions which are members of the label family with that particular variable on the left side, those with that variable appearing on the right side and so on. This searching process stops when there are no more variables to search for (LaViola & Zeleznik, 2007).

MathPad creates coordinate systems by either using the initial locations of the elements of the diagram or by labelling linear dimensions on the diagram. In case there are two different drawings associated with certain mathematical expressions and each diagram has a different value for one of the coordinates, MathPad defines an implicit coordinate system. If a numerical label is applied to a diagram, MathPad draws information from the diagram, translating a horizontal line into the x-axis and the vertical into the y-axis. If not enough information is available, MathPad uses a default coordinate system. In case the expressions do not correspond to the associated drawings, MathPad corrects the mismatches by rectifying the diagram. First it analyses the diagram itself, then determines whether a mathematical expression corresponding to drawing elements exists. If it does, it adjusts the diagram according to the expression. If not, it uses the diagram as a reference point. MathPad also entails the mathematical sketch animation feature. In order to execute the animation, the application first checks all drawings with associations to see if this is possible at all, as it is the case with for instance time functions. The mathematical expressions associated with the drawing elements in question are then sent to Matlab which calculates the expressions. All animations in MathPad last by default four seconds, but this as well can be adjusted (LaViola & Zeleznik, 2007).

With regard to user feedback after working with MathPad, the authors outline the following:

- Users expressed their wish to be able to also solve more complex problems such as double pendulum in MathPad
- Effect of misrecognition and erroneous parsing has been pointed out, which increases with the complexity of mathematical input, specifically between for instance number 5 and letter 's', letter 'x' and + etc.
- Some users readjusted their handwriting so that the parser could more easily recognise the input
- Despite the encountered problems, the authors conjecture that mathematical sketching, as handwritten mathematical expressions and diagrams, can be a powerful tool in formulating and visualising mathematical concepts (LaViola & Zeleznik, 2007)

5.2 3D software application as spatial abilities training tools by Christou et al

Several software applications have been developed as a part of the DALEST project which aim at enhancing middle school students' 3D geometry understanding and spatial visualisation skills. The developed applications focus on developing thinking and problem modelling abilities, analysing and solving problems. The goals of this project were: 1) to analyse the existing curriculum and evaluate teachers' requirements with regard to teaching stereometry, 2) develop a dynamic 3D software for teaching and learning stereometry, 3) use the developed software to teach teachers how to use it for stereometry teaching, 4) develop didactic situations with the applications, 5) test the developed software in different schools,

6) evaluate its effectiveness and teaching approaches. The project was conducted within the SOCRATES programme 2005-2006 of the EU (Christou et al, 2006).

One of the applications developed and involved in this project was Cubix Editor, which allows the user to create 3D structures built of unit cubes. The application also allows the users to recolour individual cubes and calculate volume and surface area for the created pieces. One of the distinguished characteristics of the application is the rotation option, i.e. possibility for the user to rotate the whole platform and view the created pieces from different sides and angles. This dragging capability can be used to create didactical situations in which visual abilities such as 'perceptual constancy' and 'mental rotation' are exercised (Christou et al, 2006).

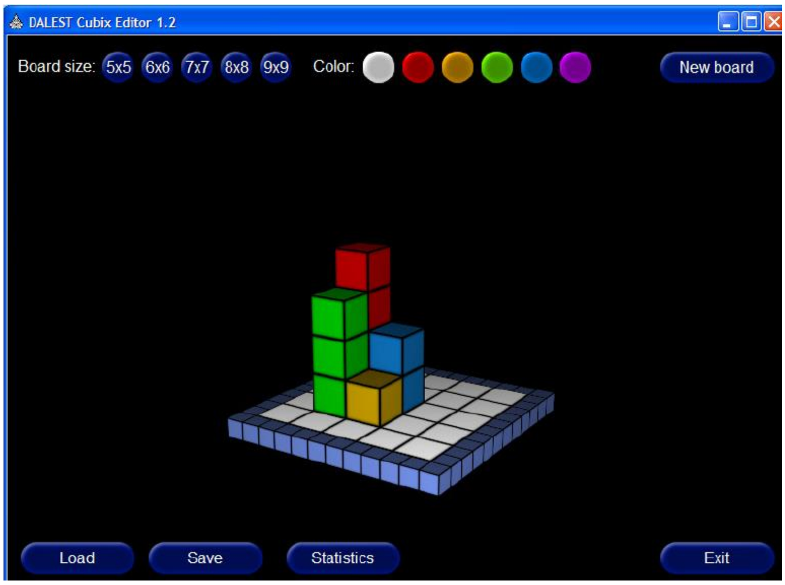

Figure 6. Environment of the Cubix Editor.

Retrieved on the 15th of January, 2017 from https://www.researchgate.net/figure/279653012_fig1_Figure-1-The-Environment-of-Cubix-Editor

Potter's Wheel is another application developed within the scope of the DALEST project, which provides the user with possibility to rotate a 2D object (five simple 2D objects are at disposal) in 3D space, which results in a 3D object. Origami Nets on the other hand allows the user to build nets using triangles, squares, rectangles and regular polygons, with defining folding angles. Transition from 2D net and 3D solid object is done fluidly and adding new elements to the already existing net is done interactively. Rotation is of course a very important feature of this application with the help of which the user can visualise how the object will look like from a different angle or side. Transition from 2D to 3D can be used to result in for instance a didactical situation in which the user tries to visualise how many times should the net be folded for it to result in a 3D object. The 2D objects available to use are: square, rectangle, triangle, regular polygon etc. The folding angle can be changed interactively and the object can be folded (3D) and unfolded (2D). There are 43 predefined nets included in the application (Christou et al, 2006).

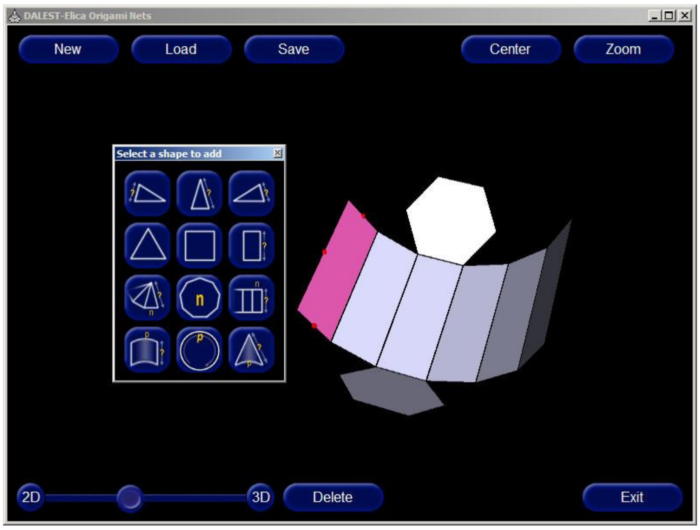

Figure 7. User interface of Origami Nets.

Retrieved on the 15th of January 2017 from http://www.elica.net/dalest/on.html

Slider is an application in which it is possible to cut through an invisible 3D solid with a plane, where only the intersection is visible. By moving and rotating the plane the user can analyse it and try to reconstruct the 3D solid. Solids can also be made visible if the user wishes to. The application offers 16 different objects in various orientations and offers more than 90 problems of various difficulty levels (Christou et al, 2006).

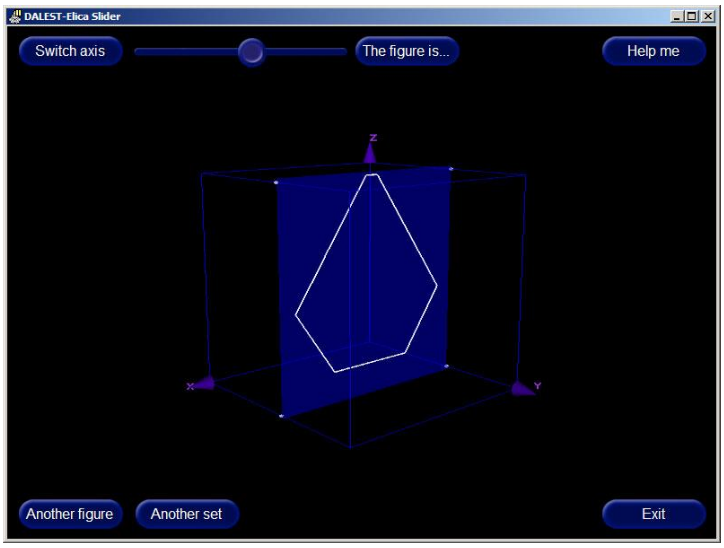

Figure 8. User interface of Slider.

Retrieved on the 15th of January 2017 from [***http://www.elica.net/dalest/sl.html***](http://www.elica.net/dalest/sl.html)

Using the 3D Cube Net application the students are able to rotate different 3D cubes nets. By opening a new window which presents eleven possible nets they can try to visualise which presented net corresponds to the one presented earlier. Scissors application on the other hand offers a given target net of a cube. The user then needs to appropriately cut a cube in order to create the net which will be the result of unfolding. This application implements the opposite process of cube net folding and users can cut along any of the edges. In case they cut less edges than it is necessary, the application will cut the rest of the edges. However if they cut more than it is necessary, it will not be possible to produce a net (Christou et al, 2006).

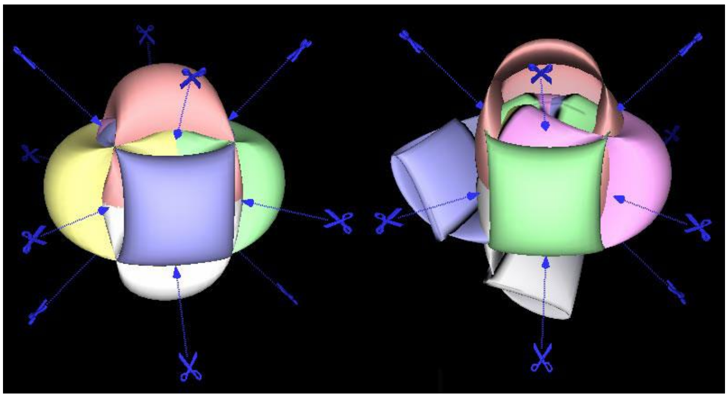

Figure 9. Scissors application.

Retrieved on the 15th of January 2017 from [***http://www.elica.net/dalest/sc.html***](http://www.elica.net/dalest/sc.html)

5.3 Development of educational software components by Roschelle et al

The authors point out the problem which arises in software development which is a lack of contact between educational software developers and students, who are the end users of the software. Their idea was to replace the developmental process with a process of editing, creating and manipulating 'editable applications'. Because of the simplicity of use, which would in some cases boil down to copying and pasting images, the teachers would have many more possibilities opening up. One possibility which such an approach offers is versatility, i.e. cross-subject use and adaptation of the same concept to different sciences, subjects or approaches. Problems which the authors anticipated were that a large library of components had to be developed and also that teachers as well as students needed to be encouraged to use the developed applications (Roschelle et al, 1999).

In order to make these obstacles more easily abridgeable, the authors decided to employ the already existing applications as component-generating tools and for that purpose relied for the greater part on the Geometer's Sketchpad and AgentSheets, which enable model export as JavaBeans. Geometer's Sketchpad was chosen because it enables constructing arbitrary geometric models which can be easily shaped and adjusted whereas AgentSheets offers creating interactive simulations, which are also very flexible. Because Geometer's Sketchpad and Agentsheets are already in widespread use and are familiar to both students and teachers, the authors have made one further step toward smoothening out the reception of application additions by the teachers and students (Roschelle et al, 1999).

Components are wired together in JavaBeans but the wiring could not remain static, since the capabilities and connectivity of a certain component may vary depending on the context in which the component is used. Since Reflection mechanism which is offered by JavaBeans employs a static analysis and which naturally is not adequate for dynamic changes, the authors employed InfoBus which offers a more dynamic analysis as well as E-Slate, which dynamically assesses a component's connection capabilities, whereas its user interface allows the user to make connections between compatible components. Two other wiring issues also needed to be addressed: the reflectivity of changes when linking components together and intracomponent issues, which refers to the fact that some of the components are rigid in their construction and adaption in some cases is hard if not impossible. While use of The

Educational Object Economy, which is a large educational Java Applet repository, might be suitable to solve the latter issue, it would require skills from teachers and students to work with Java source code. AgentSheets offers several user-programming paradigms like graphical rewrite rules, for the use of which one does not have to be a professional programmer whereas E-Slate is based on Logo, a high-level programming language used by educators. E-Slate expands Logo into an adept component scripting language which enables the user to open public scripting interface of each component and thus make it easier for the users to work with it (Roschelle et al, 1999).

In order to meet the needs of evolving software along with the curriculum and needs, the authors focused on a blend between component developers and application designers, which resulted in “Determining Component Granularity and Connectivity” sidebar, which helps identify the correct component size. Another issue the authors had to address was the fact that component developers lack the resources to create all the required components from scratch. Instead they start with models which have inadequate communication models or different communication protocols. Translators are one way of solving interoperability, along with wrappers. Translators adapts an already existing communication protocol, wrappers provide a communication layer around the component in question, which enables communication with other components. One advantage of wrappers is that they can reduce the application’s costs and risks in adapting the already existing component (Roschelle et al, 1999).

5.4 Effect of DGS software on mathematics teacher students’ spatial visualisation skills by Güven and Kosa

The aim of the authors’ study was to examine the effect of activities in Cabri 3D on the development on mathematics teacher students’ spatial skills. The study was conducted in 2007 and the sample consisted of 40 participants (22 male and 18 female) studying at the Karadeniz Technical University, Turkey. Pre-test and post-test were administered prior and posterior to the experimental programme, for which PSVT was used. The authors developed various activities for the students with Cabri 3D for at least 1.5 hours, spanning across 8 weeks. After the completion of the experimental programme, the subjects again took the PSVT as post-test (Güven & Kosa, 2008).

Week	Topic
1	Learning to work in Cabri 3D, drawing basic geometric objects
2	Obtaining point, circle, ellipse and hyperbola via cone intersections
3	Vertical projection and trigonometric relations
4	Reflection, transition and rotation in 3D space
5	Obtaining various objects and surfaces by cutting prisms, unfolding the objects
6	Estimating the folded state of objects given as unfolded in Cabri and then using the software to fold them
7	Intersecting objects with various surfaces and planes and obtaining intersection curves
8	Free exercises

Table 4. Schedule of course activities, Güven and Kosa study (Güven & Kosa, 2008).

The authors used descriptive statistics in order to have a complete picture of students' spatial skills before running the experimental programme and after it. Additionally, the authors used t-test in order to compare the pre-test and post-test scores. Compared against subtests, on PSVT:D mean value on pre-test was 6.2 whereas mean value on post-test was 7.9. On PSVT:R mean value on pre-test was 5.7 and on post-test 7.8 and on the PSVT:V mean value on pre-test was 3.8 and on post-test 5.7. Total value on pre-test was 15.7 and on post-test 21.4. T-test showed that there was a significant difference between pre-test and post-test scores, with $p<0.05$ for all subtests (Güven & Kosa, 2008).

From the pre-test results it is obvious that the students' spatial abilities are rather low, especially on the PSVT:V where the average value was 3.8 out of maximum 12, but as well on the average total score which was 15.7 out of total 36. The authors propose two possible explanations for such low results on the pre-test: 1) geometrical content is in Turkish classrooms presented in 2D format on the blackboard and therefore the students had no opportunity to create and manipulate with 3D models, which is important for developing spatial skills, 2) geometry teaching in Turkey is largely based on procedural teaching, i.e. leaning formulae by heart. The post-test showed development of subjects' spatial skills, with the biggest increase visible in the rotations sections. Cabri itself corrects the possible reasons for low spatial skills scores because it allows the students to for instance rotate 3D geometrical objects and also enables them to learn geometrical concepts and explore geometrical relationships (Güven & Kosa, 2008).

5.5 Effects of coordinate geometry learning in Geogebra on mathematics achievement by Saha, Ayub and Tarmizi

The aims of the authors' study was to: 1) examine differences in average post-test scores between control and experimental group, 2) examine differences in average post-test scores between control and experimental group among HV (high visual-spatial ability) students, 3) examine differences in average post-test scores between control and experimental group among LV (low visual-spatial ability) students. The sample consisted of 53 students (27 in experimental and 26 in the control group), consisting of two homogeneous Form Four classes in Kuala Lumpur, aged 16-17. Additionally in each group the participants were sorted out into two sub-groups, according to the level of visual-spatial ability development, low and high, thus yielding four subgroups: GGHV and GGLV (participants with high visual-spatial abilities working in Geogebra and participants with low visual-spatial abilities working in Geogebra), CGHV and CGLV (participants not working with Geogebra with high visual-spatial abilities and participants not working in Geogebra with low visual-spatial abilities). In order to be able to sort the participants into sub-groups the authors administered the SVATI (Spatial Visualisation Ability Test Instrument), which consists of Cube construction tasks, 3D spatial ability tasks and mental rotation tasks (Alias, 2000). Two weeks before starting with the experimental programme, the authors administered the SVATI (Saha, Ayub, & Tarmizi, 2010).

Work with the experimental group consisted of three phases:

1. Introduction to Geogebra. Familiarisation modules were developed in order to help the students to get familiar with the software quickly and they were encouraged to explore various features.
2. Basic concepts of the Coordinate Geometry and mathematical problems.
3. Learning Coordinate Geometry along with GeoGebra. 7 learning modules were distributed (Saha, Ayub, & Tarmizi, 2010).

After completion of the experimental programme, post-test was administered, which consisted of 6 subjection questions. Results of the independent t-test which compared post-test results of the control ($M=54.7$, $SD= 15.660$) and experimental group ($M= 65.23$, $SD= 19.202$; $t(51)=2.259$, $p=0.028<0.05$) showed that there was a statistically significant difference. Cohen's η^2 was approximately 0.09, which is a relatively small difference.

Obtained results imply that participants in the Geogebra group were significantly better in their achievements than the participants in the control group. Secondly, the authors examined the post-test results of the independent t-test comparing the results of the HV subgroup of the control group (M=61.667, SD= 13.793) and HV subgroup of the experimental group (M= 67.583, SD= 16.489; t(22)=0.953, p=0.351>0.05), which imply that there was no significant differences between mean scores. Cohen's η^2 was approximately 0.04, which is a very small difference. Lastly, the authors examined the post-test results of the independent t-test comparing the results of the LV control group (M=48.786, SD= 15.106) and LV experimental group (M= 64.067, SD= 21.569; t(27)=2.222, p=0.036<0.05). Cohen's η^2 was approximately 0.15, which is a large effect. From the obtained results it can be concluded that LV students benefited the most from the Coordinate Geometry course in Geogebra (Saha, Ayub, & Tarmizi, 2010).

5.6 Bakar et al research on teaching mathematics using Geogebra among students with dissimilar spatial visualisation

The authors' study had two aims: 1) to examine the effects of experimental programme in Geogebra on quadratic function and 2) to examine the effect of the experimental programme on students with who have different spatial visualisation abilities as a predictor of their achievements in geometry. A pre-test, consisting of questions on the quadratic functions, and a test on spatial visualisation were administered to both experimental and control group. The spatial test was used to categorise the subjects into two groups – students with high and students with low spatial abilities. After this the experimental group continued with the experimental programme in Geogebra over the following seven weeks, whereas the control group continued with the traditional, conventional teaching programme over the same time span. At the end of the seven weeks mathematics post-test was administered to both groups (Bakar et al, 2015).

ANCOVA analysis was conducted in order to determine the differences between the effects of two types of teaching, the conventional one and the experimental programme done in Geogebra, with the post-test scores as the dependent variable. Analysis showed that there was a statistically significant difference between the control and experimental group (F(1.68)=43.56; p=0.001; η^2=0.157), which implies that the students in the experimental group scored better on the post-test. Within the experimental group 12 students were sorted into the high spatial visualisation category and 23 into the low spatial visualisation category. Within the control group the division according to the level of spatial visualisation development was similar, 18 in each. Two-way ANOVA analysis showed that the differences in teaching (Geogebra and conventional) were not statistically significant for students with different spatial visualisation levels ($F(1.67)=0.582$; $p=0.448$). For the experimental group, the post-test average results were 17,50 for the high spatial visualisation group and 18.74 for the low spatial visualisation group, whereas for the control group the post-test average results were 12.89 for the high spatial visualisation group and 11.57 for the low spatial visualisation group. Viewed only with regard to spatial visualisation abilities, average results on the post-test were 14.73 for the high spatial visualisation abilities students and 15.59 for the low spatial visualisation abilities students (Bakar et al, 2015).

5.7 3DMath DGS by C.Christou, K.Jones, N.Mousoulides and M.Pittalis

The authors' aim with designing the 3DMath software was to enhance stereometry teaching in middle school, to enable the students to experiment with 3D objects. The authors argue that the way in which students are conventionally supposed to get a sense of 3D geometry from 2D is neither easy nor natural and even though available DGS can be used to construct 3D objects, most of the applications do not enable the students to develop the ability to visualise 3D objects. For the design of the application key elements of visualisation as defined by Presmeg (1986), Bishop (1980), Clements (1982) and Gutiérrez (1996) were taken into consideration (Christou et al, 2006).

3DMath enables the user to create 3D geometrical objects, which can be moved, manipulated and reshaped, which is intended to help the students to acquire and develop visualisation abilities in the context of 3D geometry. Apart from providing an exploratory environment for 3D objects, 3DMath has been designed to incorporate other content except geometry. The design of the interface is intuitive and also enables multiple perspectives and representations, e.g. the user can view the constructed 3D object also in 2D. It also enables the users to focus on the mental images they create and the process of visualisation. The application enables the user to construct different solids and then perceive them in a concrete form and view them from different perspectives, which helps them build a mental image. The design of the application also incorporates ideas of Bishop (1980), referring to the two relevant visualisation processes: 1) interpreting figural information and processing abstract information and 2) translating gathered information into visual terms, its manipulation and transformation from one mental image into another. The application enables the user to gain an insight into the concept of dimension by changing the axes, control over rotation of the constructed solid and also drawing preferences, i.e. choice between solid colours and transparent line view, atop with labelling, colouring and selecting edges and sides of the constructed objects (Christou et al, 2006).

3DMath has been designed in such a way as to enable the development of the following visualization abilities (Gutiérrez, 1996): 1) figure-ground perception, 2) perceptual constancy, 3) mental rotation, 4) spatial positions perception, 5) spatial relationships perception and 5) visual discrimination. Features of 3DMath accommodating the aforementioned abilities are 1) dragging, 2) tracing, 3) measuring and 4) history. The dragging feature enables the user to rotate, move and resize 3D objects. Objects can be resized in only one dimension or proportionally. The tracing feature refers to the instance in the design of the interface where only parts of the object are shown, which aims at visual filtering of the main construction i.e. enable the user to view parts independently. Length of edges, surface of sides but also volume of the solid can be displayed, which are designed as dynamic, i.e. as the solids are resized, all measurements follow the changes. 'History' textual feature refers to the description of the figure, which aims at providing information of all objects involved in the construction, but also the means for other users to recreate the exact same solid. Exporting constructions in various formats is possible, which enables recording work for students and also to create education material for teachers (Christou et al, 2006).

5.8 Overview of experimental programmes designed to enhance spatial abilities of STEM students

In this section a summary of experimental programmes designed to enhance spatial abilities will be presented. Some of the studies such as Bakar et al (2015) and Saha, Ayub and Tarmizi (2010) have used their entire mathematics courses as a variation of experimental programme, with both mathematics as well as spatial abilities tests administered before and after the course. Since some of the experimental courses entail a lot of mathematical exercises as tools for spatial abilities enhancement, it is in some cases difficult to differentiate between tools which serve only for spatial abilities enhancement and tasks which are a part of the actual course, given the fact that in the end results showed a statistically significant difference.

Authors	Participants	Duration	Tools
Martín - Dorta, Saorín and Contero (2008)	40 students (25 males and 15 females)	3 weeks	• Google SketchUp 5 • 24 objects made of aluminium
Sorby & Baartmans, (2000)	• 24 students (1993) • 16 students (1994) • 47 students (1995) • 26 students (1996) • 27 students (1997) • 36 students (1998)	10 weeks	• hands on construction activities • paper and pencil activities • Miras
Güven & Kosa (2008)	40 participants (22 male and 18 female)	8 weeks	• Cabri 3D
Bakar et al, 2015	35 (experimental group), 36 (control group)	7 weeks	• Geogebra
Saha, Ayub, & Tarmizi, 2010	53 students (27 in experimental and 26 in the control group)	Whole course	• Geogebra

Table 5. Overview of experimental programmes with outlined authors, participants, duration and employed tools

In the following table the actual content of experimental programmes as well as study results have been outlined, thus completing a picture of the studies done worldwide which focused on creating experimental programmes or courses aiming at enhancing spatial abilities. The duration of programmes varied from 3 weeks to whole courses, as well as the tools used, with paper and pencil method, Google SketchUp 5, Miras, Cabri 3D and Geogebra. Since positive correlation has been found in all the outlined studies, these studies can be considered as positively corroborated tools for spatial abilities enhancement, despite of the fact that the duration, the employed tools, participants and actual content of the programme widely varied.

Authors	Content	Results
Martín ‐ Dorta, Saorín and Contero (2008)	• Building 3D models based on the appearance of the aluminium objects • Creating a daily object • Creating 3D models corresponding to parts given by their axonometric projection • Constructing 3D models of parts which are represented by their orthographic views • Group work	Gain differences on the MRT test was 5.48, whereas on the DAT:SR was 8.30, regardless of gender (*t*-test for independent series *p*= 0.552 and *p*= 0.868)
Sorby & Baartmans, (2000)	• isometric and orthographic sketching • orthographic drawings and applications • pattern development • Two and three coordinate drawing • Translation and scaling • Object rotation • reflection of objects and applications • Cross-sections of solids • surfaces and solids of revolution and the intersection of solids	• 1993: gain 9.17, t-value 12.5, p<0.0001 • 1994: gain 9.69, t-value 10.7, p<0.0005 • 1995: gain 6.66, t-value 12.2, p<0.005 • 1996: gain 9.42, t-value 9.42, p<0.0005 • 1997: gain 8.89, t-value 9.91, p<0.0005 • 1998: gain 6.53, t-value 10.46, p<0.0005
Güven & Kosa (2008)	• Learning to work in Cabri 3D, drawing basic geometric objects • Obtaining point, circle, ellipse and hyperbola via cone intersections • Vertical projection and trigonometric relations • Reflection, transition and rotation in 3D space • Obtaining various objects and surfaces by cutting prisms, unfolding the objects • Estimating the folded state of objects given as unfolded in Cabri and then using the software to fold them • Intersecting objects with various surfaces and planes and obtaining intersection curves • Free exercises	• Total value on pre-test was 15.7 and on post-test 21.4 • T-test showed that there was a significant difference between pre-test and post-test scores, (p<0.05)
Bakar et al, 2015	Same course in mathematics, with the difference that experimental group	statistically significant difference between the

	worked in Geogebra and the control group worked using traditional teaching methods	control and experimental group ($F(1.68)=43.56$; $p=0.001$; $\eta^2=0.157$)
Saha, Ayub, & Tarmizi, 2010	• Introduction to Geogebra. Familiarisation modules were developed in order to help the students to get familiar with the software quickly and they were encouraged to explore various features. • Basic concepts of the Coordinate Geometry and mathematical problems. • Learning Coordinate Geometry along with GeoGebra. 7 learning modules were distributed	independent t-test compared post-test results of the control ($M=54.7$, $SD= 15.660$) and experimental group ($M= 65.23$, $SD= 19.202$; $t(51)=2.259$, $p=0.028<0.05$)

While the study done by Martín ‐ Dorta, Saorín and Contero (2008) lasted only three weeks, pre-graphics course designed to enhance spatial abilities by Sorby and Baartmans (2000) spanned throughout 10 weeks. The two studies which did not create an experimental programme explicitly for the purpose of spatial enhancement but rather opted to use GeoGebra with the already existing mathematical course content done by Bakar et al, 2015 and Saha, Ayub, & Tarmizi (2010) nevertheless showed positive correlation between the pre-test and post-test in spatial abilities. Both studies employed GeoGebra, which software is known for its ease of use and also applicability in mathematical courses. Güven & Kosa (2008) study employed Cabri 3D with mostly mathematical content, whereas Martín ‐ Dorta, Saorín and Contero (2008) focused mainly on the content applicable for students of engineering (orthographic view, sketching 3D models).

Despite of differences in content and duration all these studies need to be taken into account when creating an experimental programme aiming at enhancing spatial abilities, since results clearly show their significance.

6 GeoGebra

GeoGebra was developed as a part of Markus Hohenwarter's master's thesis, entitled *GeoGebra – a Software System for Dynamic Geometry and Algebra in the Plain* (in original: *GeoGebra - Ein Softwaresystem für dynamische Geometrie und Algebra der Ebene).* Hohenwarter got the idea for the software by analysing the applications which were available: the CAS, Computer Algebra Systems such as Maple and Mathematica and DGS such as Cabri, Cinderella and CAD. According to the author, the DGS applications enable the user to draw geometrical objects and manipulate them, but the algebraic input is limited in such a way that with some of these applications it is e.g. possible to see the equation describing the shown geometrical object, but the equation itself cannot be manipulated or changed by the user. The CAS applications on the other hand enable the user for instance to change the equations and also draw the objects described by the equations, however the geometric representation cannot be changed. After analysis of advantages and disadvantages of the available applications, the author intended to connect these two types of software, in a way that each object would be shown in two ways: algebraically and geometrically. When the user manipulates the geometrical representation in some way (shifting, rotating, reflecting), the changes in the geometrical representation are instantly visible in the algebraic representation, but also vice versa. Such an approach to the development of the application was aimed at resulting in a close connection between algebra and geometry (Hohenwarter, 2002).

Since its development, GeoGebra received several international software awards: European Academic Software Award 2002 (Ronneby, Sweden), L@rnie Award 2003 (Vienna, Austria), digital 2004 (Cologne, Germany) and Comenius 2004 (Berlin, Germany) (Hohenwarter & Fuchs, 2004).

6.1 General GeoGebra features and GeoGebra 5 novelties

As interactive geometry software, GeoGebra has been designed primarily for students and aimed at being a mathematics learning tool, approaching it in an experimental way. The dragging feature of GeoGebra enables the user to investigate changes in the geometrical object or shape while following the changes in the algebraic representation, thus investigating how e.g. parameters of a circle equation change. Changes are possible at any point, including inserting new elements and also changing the order (Hohenwarter & Fuchs, 2004).

GeoGebra offers four different views, displayed in different representations which are dynamically connected: algebra, graphics, CAS, 3D graphics and spreadsheet view. Graphics and algebra view are opened simultaneously by default. Using the input field or the input bar, for which also a wide range of commands is available, it is possible to directly enter algebraic expressions, with the graphic representation automatically displayed in the graphics view. If 3D graphics view is selected, again algebra view is opened simultaneously by default. Spreadsheet view enables the user to work with data and statistics, and is opened by default alongside of the graphics view. It is also possible to import data in the spreadsheet view in GeoGebra, either by simply copying and pasting or by using import data feature (GeoGebra Manual, https://wiki.geogebra.org/en/Manual).

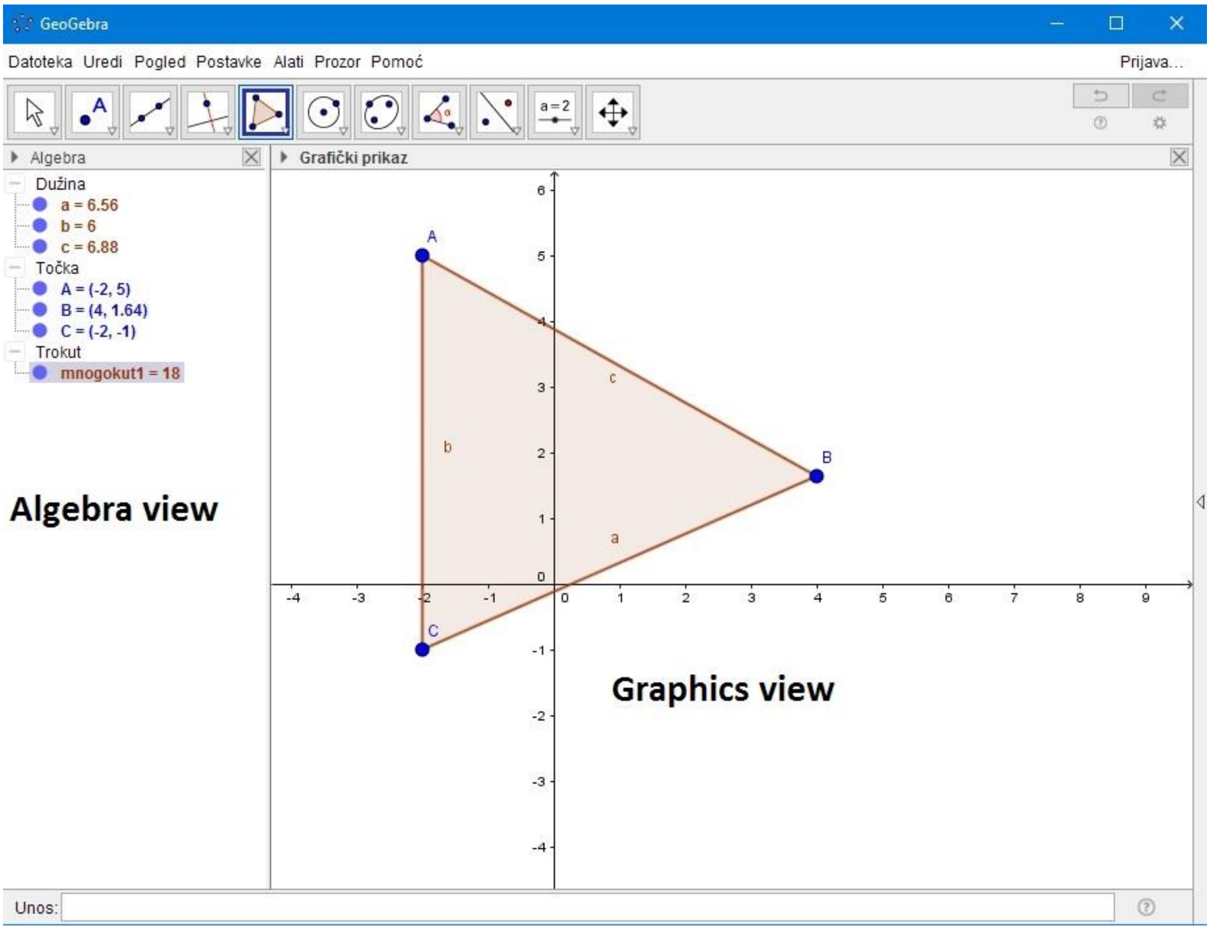

Figure 10. GeoGebra algebra and graphics view.

Point coordinates, vector coordinates, equations etc. are all visible in the algebraic window, whereas the graphic window mirrors the changes in the algebraic input simultaneously, in which also text input is possible. Toolbar is placed directly under the menu bar, where not all buttons are visible straight away, but are offered in a dropdown menu (GeoGebra Manual, https://wiki.geogebra.org/en/Manual).

Graphics tools are sorted in several groups:

Movement Tools: 1) Move tool, 2) Move around Point Tool.

Point Tools: 1) Point Tool, 2) Point on Object Tool, 3) Attach/Detach Point Tool, 4) Intersect Tool, 5) Midpoint or Center Tool, 6) Complex Number Tool, 7) Extremum Tool, 8) Roots Tool.

Line Tools: 1) Line Tool, 2) Segment Tool, 3) Segment with given length tool, 4) Ray Tool, 5) Polyline Tool, 6) Vector Tool, 7) Vector from Point Tool.

Special Line Tools: 1) Perpendicular Line Tool, 2) Parallel Line Tool, 3) Perpendicular Bisector Tool, 4) Angle Bisector Tool, 5) Tangents Tool, 6) Polar or Diameter Line Tool, 7) Best Fit Line Tool, 8) Locus Tool.

Polygon Tools: 1) Polygon Tool, 2) Regular Polygon Tool, 3) Rigid Polygon Tool, 4) Vector Polygon Tool.

Circle and Arc Tools: 1) Circle with Centre through Point Tool, 2) Circle with Centre and Radius Tool, 3) Compass Tool, 4) Circle through 3 Points Tool, 5) Semicircle through 2 points Tool, 6) Circular Arc Tool, 7) Circumcircular Arc Tool, 8) Circular Sector Tool, 9) Circumcircular Sector Tool.

Conic Section Tools: 1) Ellipse Tool, 2) Hyperbola Tool, 3) Parabola Tool, 4) Conic through 5 Points Tool.

Measurement Tools: 1) Angle Tool, 2) Angle with Given Size Tool, 3) Distance or Length Tool, 4) Area Tool, 5) Slope Tool, 6) Create List Tool.

Transformation Tools: 1) Reflect about Line Tool, 2) Reflect about Point Tool, 3) Reflect about circle Tool, 4) Rotate around Point Tool, 5) Translate by Vector Tool, 6) Dilate from Point Tool.

Special Objects Tools: 1) Text Tool, 2) Image Tool, 3) Pen Tool, 4) Freehand Shape Tool, 5) Relation Tool, 6) Function Inspector Tool.

Action Object Tools: 1) Slider Tool, 2) Check Box Tool, 3) Button Tool, 4) Input Box Tool.

General Tools: 1) Move Graphics View Tool, 2) Zoom in Tool, 3) Zoom out Tool, 4) Show/Hide Object Tool, 5) Show/Hide Label Tool, 6) Copy Visual Style Tool, 7) Delete Tool (GeoGebra Manual, https://wiki.geogebra.org/en/Manual).

The input bar, which is normally positioned at the bottom of the interface, is not displayed by default in the Algebra View, however can be shown using the View Menu. The input bar allows the user to create and redefine mathematical objects, but also allows command input, which follows a simple syntax. For instance in order to yield all local extrema of a function, the command *Extremum [ ]* is given. After entering each command it is required to hit Enter key. Some of the syntax, e.g. indices, follow the LaTeX syntax (GeoGebra Manual). All available commands are displayed in the dropdown menu on the right bottom side. GeoGebra requires the user only to start typing the beginning of a command, and then offers the correct syntax which is accepted again by hitting the Enter Key (Šuljić, 2005).

Context Menu is opened by right clicking the object and offers access to advanced properties of an object (e.g. polar or Cartesian coordinates). Navigation bar offers a set of buttons and displays the number of construction steps. It enables the user to move from one step to the other, going forward or backward and also play the construction (GeoGebra Manual, https://wiki.geogebra.org/en/Manual).

Novelties in GeoGebra 5 feature new Tools: Plane, Right Prism, Sphere and View in front of Tool, with Freehand Shape Tool improved to recognize ellipses. Points, vectors, lines, segments, rays, polygons and circles got extended to 3D, whereas new object types feature e.g. planes in 3D, also geometrical objects like pyramids, prisms, spheres, cylinders and cones. Moreover, moving objects in 3D, translation, rotation and zoom in 3D have been introduced. Many new commands have been developed or improved, along with commands for work in 3D (GeoGebra Manual, https://wiki.geogebra.org/en/Manual).

6.2 GeoGebraWeb, GeoGebraTube , Smartphones App, GeoGebraWiki

Since 2009 the desktop version of GeoGebra and the mobile version have been two different applications with two separated source code trees. This has been done in order to explore whether it was feasible to make the GeoGebra applets work without Java plugins, based only on HTML5. In 2011 the developers decided to split the GeoGebra desktop code into the "common" project and a subproject "desktop" for the GUI. The former GeoGebraMobile would get its own subproject, but relying on the "common" part, without regard to the "desktop" subproject. The subproject on the former GeoGebraMobile was developed and called GeoGebraWeb (Ancsin, Hohenwarter & Kovacs, 2013).

The so-called dynamic worksheets, interactive HTML pages, can be created in GeoGebra, for which the application itself does not need to be installed in order to use the worksheet. The worksheets however used to be dependent on use in browsers which support Java. The application can be run on any platform (Win, Unix, Linux, MacOS) and can be run via internet (Hohenwarter & Fuchs, 2004). The GeoGeobraTube was started in September 2011 as a place for sharing GeoGebra worksheets and resources. GeoGebraTube has been using the GeoGebraWeb viewer as a way to present interactive worksheets to devices not supporting Java applets since March 2012, such as iPads, Chromebooks or Android tablets. Compatibility between GeoGebraWeb and GeoGebra desktop version is ensured through their sharing of the "common" source code. Through the desktop GeoGebra it is possible to upload material to the GeoGebraTube easily, where the GeoGebraWeb enables easy use of the uploaded material, non-dependant on the device used. In May 2012 the authors published a first version of GeoGebraWeb in Chrome Web Store, which enables the creation of new constructions without the need of Java plugin, where algebra view as well as input bar are available, with many languages supported. Sharing worksheets and files is possible through saving them on GeoGebraWeb to Google Drive and then sharing through Gmail or Google+ (Ancsin, Hohenwarter & Kovacs, 2013).

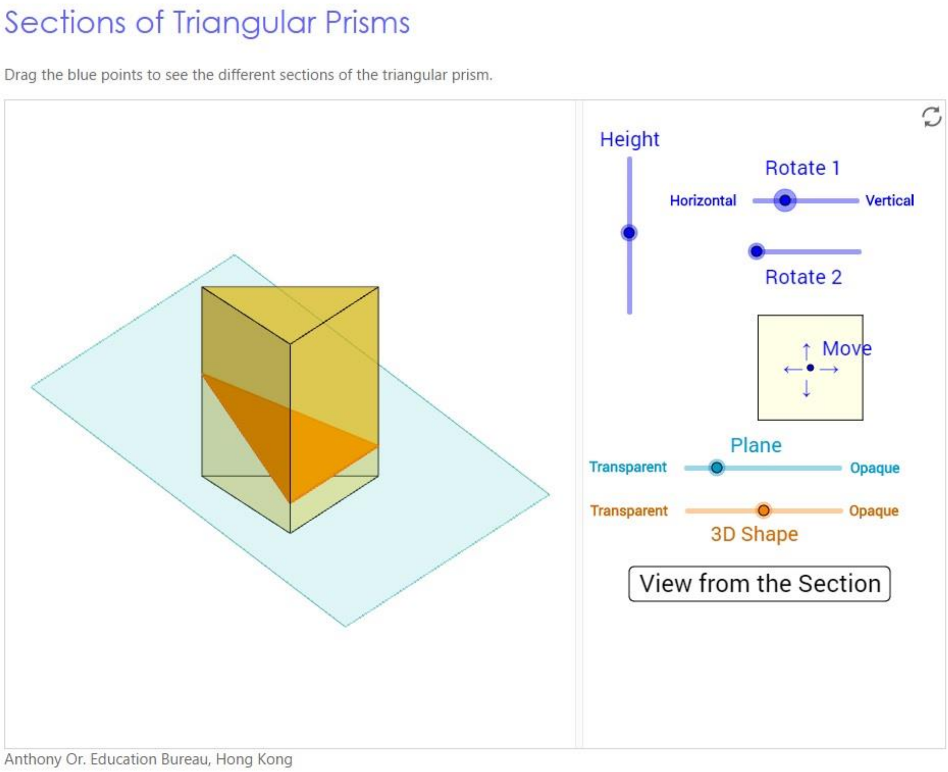

Figure 11. GeoGebraTube.

Retrieved on the 8th of February, 2017 from https://www.geogebra.org/m/xwGbTuzE

Creating a GeoGebra app for smartphones was a project which needed to unite the special features and requirements of GeoGebra and the limited resources of smartphones. The screen size is different and the hardware components are different, as the processor speed and storage capabilities are different on smartphones. The developers had to meet all this criteria while developing the GeoGebra app for smartphones (Tomaschko & Hohenwarter, 2016).

The app is available since December 2015 in Google Play Store. Once it is installed and opened, an empty GeoGebra file is shown with graphics and algebra window open. The GeoGebra tool list is shown on the upper left side when tapped on the edit symbol. The created objects can then be manipulated, moved or their properties changed by touching the screen. A single virtual keyboard is offered for the input of algebraic expressions, specially adapted for input of mathematical formulae. Additionally the formula editor is offered which checks the input of GeoGebra commands and mathematical expressions. It is possible to save one's work after login with the GeoGebra account and also to search for available materials, which is shown in size adapted to screen (Tomaschko & Hohenwarter, 2016).

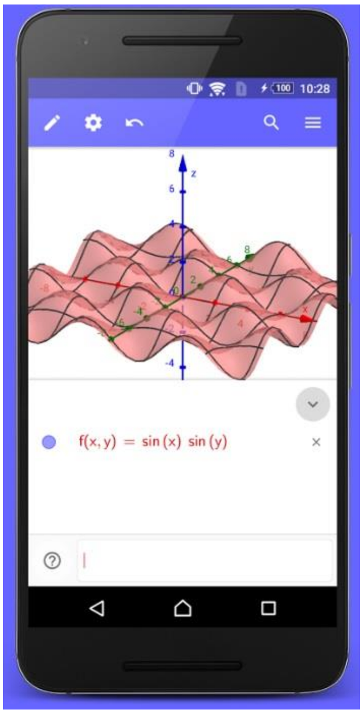

Figure 12. GeoGebra 3D App.

Retrieved on the 10th of January from https://play.google.com/store/apps/details?id=org.geogebra.android.g3d

Due to the suggestions of many teachers, GeoGebraWiki has been created in 2005 as a platform for exchange of teaching materials done in GeoGebra. Every user can upload material via GeoGebra Upload Manager and link it with the appropriate GeoGebra Wiki site. Before 2011 there were three problems with uploading worksheets: 1) in order to link the materials to a GeoGebra Wiki site, appropriate commands needed to be entered directly into the source code, 2) searching tool for materials was very limited in its capabilities, 3) material exchange itself was rather inconvenient due to the complexity of the upload process – first a HTML website had to be created with GeoGebra, the materials had to be uploaded using the GeoGebra Upload Manager and thirdly the materials needed to be linked with appropriate sites in GeoGebra Wiki (Gassner & Hohenwarter, 2012).

GeoGebraTube was created in 2011 as an alternative to GeogebraWiki. Its advantages were: 1) direct worksheet upload (the user needed to click on "File" and then "Publish"), 2) the searching tool was improved, and worked on the basis of metafiles and keywords, 3) easy way to rate the uploaded material and post comments. Additionally, the worksheets can be shared on Google+, Facebook, Twitter or simply email, thanks to the “embed-code” (Gassner & Hohenwarter, 2012).

In 2007 the developers established the International GeoGebra Institute, in order to support and assist the rapid growth of the GeoGebra community. The aims of the Institute are to offer training and support of educators, develop teaching materials and the software itself, conduct research and reach out to communities which are less well-off. The IGI acts as an umbrella association which unites local GeoGebra institutes across the world, established by teachers and researchers, focusing on their own needs, interests and priorities. Local institutes can post their meetings and workshops on the GeoGebra website and specialised GeoGebra conferences across the world are organised. The local institutes help with training and supporting teachers in their communities and are also involved in projects, locally but also globally important (Hohenwarter & Lavicza, 2010).

6.3 Advantages of GeoGebra

The advantages of GeoGebra and its versions have been summed up by various authors as follows:

- Allows school notation input (Hohenwarter & Fuchs, 2004)
- Independent of the platform and also does not need to be even installed on the device (Hohenwarter & Fuchs, 2004)
- Parallel running, dynamic graphics and algebra window (Hohenwarter & Fuchs, 2004)
- Managing jumping objects: in Cabri when an ellipse becomes a hyperbola the intersection points for two conic sections can permutate, whereas in GeoGebra this continuity problem is managed via a heuristic approach to eliminating jumping objects (Hohenwarter & Fuchs, 2004)
- Very user-friendly. It offers an interface which is easy to use, multilingual menus, commands and help files (Palve, 2016)
- Encourages students to create mathematics projects, make presentations and engage in experimental learning. Because variables can be manipulated by dragging free objects or using sliders and easy changes to free objects can be made through which

process the students can learn how the dependant objects are affected by the changes, the students are given the opportunity to solve problems by experimenting with mathematical relations (Palve, 2016)

- Personalisation of constructions and projects is possible (e.g. font size, graphics quality, colours, line thickness, line style etc.) (Palve, 2016)
- GeoGebra offers a good opportunity for work in groups, interactive class teaching or individual/group students' presentations (Palve, 2016).
- Worksheets can be easily published (Palve, 2016)
- GeoGebra encourages teachers to use it as a tool for visualisation and investigation in mathematics and also for interactive mathematics classes (Palve, 2016)
- GeoGebra documentation is reliable and easily understandable, there is a myriad of resources available on GeoGebraTube and also support on forums is quick and expert (Morphett, Gunn & Maillardet, 2015)

6.4 Criticism of GeoGebra

Criticism and deficiencies of GeoGebra have been summarised by authors in the following way:

- GeoGebra commands can prove to be a challenge for students who do not have programming experience (Palve, 2016)
- For some students independent work and experimenting with GeoGebra may not be appropriate (Palve, 2016)
- GeoGebra does not have a built-in animation modules (Palve, 2016)
- Research on how GeoGebra affects teaching and mathematics learning is limited (Palve, 2016)
- Poor performance when running applets in HTML mode, especially for larger applets or complex animations, which varies with hardware and software configuration (Morphett, Gunn & Maillardet, 2015)
- Software bugs with regard to applets which refer to e.g. function not working as expected or constructions which cause an application crash (Morphett, Gunn & Maillardet, 2015)

- GeoGebra's mathematical functions library is not as extensive as it might be required (Morphett, Gunn & Maillardet, 2015)

6.5 GeoGebra in education in general and in the Republic of Croatia

Generally, the role of mathematical software in teaching mathematics is certainly an important one and Schneider (2002) summarises four aspects that mathematical software can offer to the process of mathematics teaching (Schneider, 2002):

1) *multiple display options*: availability of different display ways of mathematical content, along with gradual transition from one display to the other

2) *experimental work*: students being able to experiment in software in order to gain new knowledge, ideas and problem solving approaches

3) *elementarisation of mathematical methods:* software enabling the use of elementary methods which have been abandoned due to the complex calculations

4) *modularity:* software enabling to directly invoke commands and not have to bother with algorithms or calculation methods (Schneider, 2002).

According to Hohenwarter and Fuchs (2004), GeoGebra can be used for mathematics classes in different ways:

1. Demonstration and visualisation: GeoGebra has a wide range of coverage because it offers various representations
2. Construction tool: Karl Fuchs pointed out on 1990 the importance of blending traditional methods and computer aided drawing in teaching geometry, for which purpose GeoGebra satisfies all criteria.
3. Discovering mathematics using GeoGebra, and giving the students the opportunity to organise knowledge on their own.
4. Preparing teaching materials: GeoGebra encourages the teachers to use it for preparing educational material as a cooperation, communication and representation tool (Hohenwarter & Fuchs, 2004).

As stated before, GeoGebra can be used as tool for experiments or to clarify or present certain concepts to the whole class. Its developer, Markus Hohenwarter (2004), summarises the purposes of GeoGebra as a presentation and experimentation tool in the following way:

- Start with an empty desktop and then create a construction in front of the class
- Load parts of a construction with appropriate additions
- Use an already completed construction, where it is also possible to choose to execute the construction step-by-step (Hohenwarter, 2004).

GeoGebra is also available in Croatian language and since it is an open source application, it is easily accessible to students. Indeed Šuljić (2005) finds that GeoGebra has grown in popularity in the Republic of Croatia because, among other reasons, it is freeware software. For other reasons for its popularity he gives (Šuljić, 2005):

1) It is a professionally made programme, which has won many European software rewards (including those for educational software)

2) It is available in Croatian.

3) It covers mathematical programmes for elementary and secondary schools in the Republic of Croatia

4) Is able to bring geometry and algebra closer than any other programme

5) Entails an intuitive algebraic equation input

6) It is easy to use for both teachers and students

7) A student can use it since the fifth grade of elementary school until he graduates from secondary school

8) It has high-quality graphics, suitable especially for classroom projections

9) Applet construction and publishing it are easy to do with GeoGebra

10) The constructions can be exported to other presentations or programmes, including LaTEX (Šuljić, 2005).

Regarding the use of GeoGebra in the Republic of Slovenia, Lipovec and Zmazek warn against using GeoGebra applets as teaching materials, arguing that solving a mathematical problem in GeoGebra during class also serves as a teaching lesson in how to use GeoGebra. Authors furthermore argue that GeoGebra applets are more suited for homework or independent research during class (Lipovec & Zmazek, 2014). Similarly, a web portal called E-um for teaching and learning mathematics, focusing primarily on the mathematical curriculum in elementary schools and secondary education had been developed between 2006 and 2009. With materials continuously being added to the portal, it contains over 1200 e-learning materials for mathematics lessons and among other content, also GeoGebra applets (Repolusk, 2009).

7 Framework for spatial abilities training in GeoGebra

7.1 Introduction

One of the studies which inspired the developed experimental programme was done by Martin-Dorta, Saorin and Contero (2008), who created a spatial skills training remedial course for engineering students studying civil engineering at the University of La Laguna in Spain, for which they used physical objects and Google SketchUp5. After getting familiar with the program, the subjects were required to create a 3D model of an aluminium object they were given by first sketching it on paper and then in software. The second research phase entailed students creating 3D models which corresponded to the parts given by their axonometric projection. During the last phase the subjects needed to construct 3D models of parts given their orthographic views. The authors administered the MRT and DAT:SR as pre- and post-tests. The results showed gain differences on the MRT test (5.48) and on the DAT:SR (8.30) (Martín-Dorta, Saorín, & Contero, 2008).

Sorby and Baartmans (2000) developed a course in 1993 on the MTU with the purpose of improving the spatial skills of freshmen. Before starting with the course the authors administered the PSVT:R, MRT, MCT and DAT:SR. During first week the students were introduced to the need for visualisation skills in STEM fields. In second week the students were given coded plans and a set of cubes and were instructed to construct a building with the cubes according to the plans. Classes in third week entailed making orthographic and isometric sketches of objects containing inclined surfaces. During classes in fourth week the subjects were working with unfolded patterns which needed to be folded into 3D. In fifth week the students learned 2D and 3D coordinate drawing – students learned how to locate specific points in space and use a table a coordinate date in order to draw their models. In sixth week 3D transformations were introduced and it was required of the students to draw objects after translation and scaling. Classes in seventh week entailed working with objects made of snap cubes and after rotating them to sketch isometric views of the objects. During the course of the eighth week it was required of the students to construct reflected images, for which purpose Miras was used, whereas during week nine the students worked with cross-sections of solids. In week ten it was required of the students to sketch the surfaces of solids which were the result of rotation of planar figures around an axis and vice versa (Sorby & Baartmans, 2000).

On the sample consisting of 535 subjects (418 male and 117 female) the PSVT:R was administered at the beginning of the year and the average percent of correct answers on the test was 79.6% for males and 68.1% for females, along with a background questionnaire. While on the pretest the students scored an average of 15.5%, on the post-test the students scored a mean test score of 24.7% (Sorby & Baartmans, 2000).

Güven and Kosa (2008) conducted a study in 2007 on the sample consisting of 40 participants (22 male and 18 female) studying at the Karadeniz Technical University in Turkey. The authors administered pre-tests and post-tests, which consisted of PSVT:R, PSVT:D and PSVT:V, prior and posterior to the experimental programme. The experimental programme itself ran throughout the course of eight weeks. The authors developed various activities which were done in Cabri 3D for at least 1.5 hours weekly. During the course of week one the students learned how to work in Cabri 3D and how to sketch basic geometric objects. During week two the students learned how to obtain point, circle, ellipse and hyperbola by using cone intersections whereas in week three they worked with vertical projection and trigonometric relations. In week four various rotation, reflection and transition exercises were done in 3D. In week five the students learned to obtain different surfaces and objects by cutting prisms whereas in week six they worked with unfolded objects in Cabri, estimating the appearance of the object when folded and vice versa. During week seven the students learned to obtain intersection curves by intersection objects with surfaces and planes and work in week eight comprised of free exercises (Güven & Kosa, 2008).

On PSVT:D the mean value was 6.2 on the pre-test and 7.9 on the post-test, on PSVT:R the mean value on the pre-test was 5.7 and 7.8 on the post-test whereas on PSVT:V the mean value on the pre-test was 3.8 and on the post-test 5.7. On the whole PSVT the mean value for pre-test was 15.7 and on the post-test 21.4, with t-test showing a statistically significant difference between the pre-test and post-test scores ($p<0.05$) (Güven & Kosa, 2008).

The described studies aimed at improving students' spatial abilities. Some of these studies lasted for a whole semester, whereas some were designed as a shorter course. The author's developed experimental programme has been designed to run across four weeks, adapted to the university at which it was being held, its resources and offered options. The exercises done were also adapted to the time given with the students and also to their pace of learning and applying knowledge. Introductory lesson during which the subjects learned to work in GeoGebra has been necessary since the subjects' experience with working in mathematical software in general has been minimal and insubstantial to match the relative complexity of the exercises in the experimental programme.

The wooden models created and used in our experimental programme have not been as elaborate and complex as those used by Martin-Dorta, Saorin and Contero (2008) because of the following reasons: 1) the subjects were not engineering students and their previous experience with working with sketching objects was minimal, which is why it was decided to use less complex objects, 2) due to the time frame given for the experimental programme which was limited, it was decided to focus on work with wooden objects during the course of a single lesson.

With regard to the exercises done in third week of the experimental programme it must be pointed out that while it would also be valuable to work with rotation and reflection in 3D, 2D was more applicable and feasible in GeoGebra. While the students could easily mark points in 2D, connect them using the 'polygon' tool and then check their solution using the appropriate tool, the current version of GeoGebra does not yet support connecting points in 3D space in order to form a geometrical object which is why the exercises were designed for 2D only.

During the lessons in week four of the experimental programme various mathematical problems have been presented with emphasis on visualisation. Again due to the limited time frame planned for this lesson a couple of mathematical problems have been chosen and presented to the subjects, which have been in turn split into two levels: level one (basic) and level two (advanced). While it was expected of some subjects to show proficiency in solving mathematical tasks of this sort, who would also have no problems with visualisation, it was likewise expected of some subjects to struggle with solving the problems, which is why the tasks have been split into two levels, what is in line with methodical approach to teaching any mathematical content.

It must be pointed out that the whole programme was conducted within limited time frames. It was hoped however that in the future a similar experimental programme could be conducted, based on the same principles and featuring similar content, but spanned over a longer period and going more in depth with the content and the exercises. Likewise it is expected that a more elaborate version of the experimental programme would yield better results.

Experimental programme with the predicted duration of four weeks has been designed for the experimental group, which has been divided into two equal groups. It has been beforehand decided that the experimental groups should not be larger than 20-25 students in order for the teacher to be able to control the correct execution of all steps. It has also been concluded that the experimental programme would be presented to 50-60 students, but not less than 35. The reason for such a small sample lies in the fact that the exercises would not just be handed out by the researcher and then the students' work examined upon completion of each exercise, but that the exercises need to follow a specific number of steps:

- First step: the student needs to visualise the problem given.
- Second step: the student needs to sketch it using pen and paper method.
- Third step: the student sketches the problem in Geogebra.
- Fourth step: advanced exercises and individual analysis of similarities and differences between the computer and paper sketches for the purpose of advancing their spatial abilities, which has been left to the subjects.

These four steps aim at students first creating a mental image of the problem given, sketching their mental image using pen and paper method and only then using software to sketch the problem so that they could compare what they visualised before and the image they got sketching in GeoGebra. It is hypothesised that the method of first visualising the problem and then confirming their mental image in software would be a great aid in the whole process of spatial abilities training.

Upon the completion of the programme every student and his individual progress were analysed, as well as the initial and final testing, which was done with the control group at the same time as with the experimental one.

The number of subjects needed to be much larger than the expected valid sample size because the experimental programme has been conducted throughout the course of four weeks and it was expected that some students would not attend all of the sessions and would have to be eliminated from the research.

The control group was a parallel group of students also in their first year and studying at the same university as the experimental group but they did not attend the experimental programme. The control group simply continued with their usual activities and were not subjected to any type of influence whatsoever. In a nutshell both students sorted in the experimental and in the control group studied at the same university, were of the same age and came approximately with the same knowledge from high schools (as judged by the entrance exams). The only difference between them was the fact that half of them participated in the experimental programme and the other half did not.

In the following few sections it will be outlined which type of exercises had been done with subjects during classes. The framework itself has been designed flexibly in the sense that the exercises could be adapted and extra GeoGebra materials could be used, taking into consideration the students' capabilities and technical/time management details.

7.2 First week

Work undertaken in the first week was introductory work in GeoGebra 5, i.e. learning how to use for instance 3D and 2D view, how the input field functions and the syntax required. After the subjects have familiarised themselves with basic commands in GeoGebra, it was proceeded to first visualising and then sketching geometrical objects, which aimed at serving as the opportunity to apply the basic knowledge they have gained during this introductory class.

Exercise 1. Draw a line through two points A=(1,2,3) and B=(3,1,2).

Method 1: Click on "point", establish x and y coordinates, then drag upwards in order to define the z coordinate. While doing so the students could follow in the algebra window what was happening. In case they defined the coordinates wrong, they could right-click the point in the algebra window, click on "properties" and redefine the coordinates.

After this has been done, the students needed to click on “line through two points” and click on one of them. Once the line appeared, they needed to position it to run through the second point and then click again. Now the line equation was visible in the algebra window.

Method 2: The students could also enter commands in the input field:

A=(1,2,3)

B=(3,1,2)

Line[A,B]

<u>Exercise 2.</u> Sketch a sphere with the centre in point C=(4,1,3) and diameter 6.

Method 1: First, 3D view needed to be activated. Once this has been done, the students could simply click on “Sphere” button, define the centre by clicking on the point after what GeoGebra asked for radius.

Method 2: The students could also enter commands in the input field:

C=(4,1,3)

Sphere[C,3]

After they have completed this task they have been asked to use the “rotate” tool in order to view the sphere from different angles.

<u>Exercise 3.</u> Sketch a regular square pyramid, which has for its base vertices (2,2,0), (2,-2,0), (-2,2,0) and (-2,-2,0). The height of the pyramid is 6.

Method 1: After 3D view has been activated, the students could click on the “pyramid” button and it would be requested of them to click on the vertices of the base in 3D view and then choose the apex of the pyramid.

Method 2: The students could also enter commands in the input field:

A=(2,2,0)

B=(2,-2,0)

C= (-2,2,0)

D=(-2,-2,0)

Pyramid[poly1, 6]

After they have completed this task they were asked to use the “rotate” tool in order to view the pyramid from different angles.

Exercise 4. Sketch a circle with radius 5 which lies on the y-z plane, having the point (0,3,3) as its centre.

Step 1/2. The students were asked to visualise the described circle and then try to sketch it on a piece of paper.

Step 3. Sketching in GeoGebra. It was on the students to decide which of the working methods they wanted to use, based on their preference. Roughly half of the students decided on the first and the second half on the second method. After they sketched the circle in GeoGebra, they were instructed to rotate it to see it from different angles.

Commands (for those who decided to use this method):

A=(0,3,3)

Circle[A,5,x=0]

Step 4. The following question has been asked and it was requested of them not to consult GeoGebra for answers for the time being:

> If we are sitting in a point on z axis high above of the circle, which shape would it assume?

The students were left to visualise and think about it for a while. In both groups approximately 10% of them answered the question correctly (the correct answer was that it would look like a segment), whereas the rest did not know the answer. After this they were asked to use the “rotate” tool in order to see in GeoGebra what was the correct answer.

Exercise 5. Sketch a cone which has for its base a circle with radius 3, lies on a plane which is parallel to the xy plane, whereas the distance from it is 3 and the height of the cone is 5.

Step 1/2. The students were asked to visualise the described cone and then try to sketch it on a piece of paper.

Step 3. Sketching in GeoGebra. It was left to the students to decide which method to use in order to sketch the cone and no instructions regarding a specific method were given. It was also left to the students to decide which point should be used as the circle centre. The students were instructed to use the rotate tool once they were done with their work in order to see the cone from different angles.

Step 4. The following questions were asked and it was requested of them not to consult GeoGebra for answers for the time being:

1. If we are sitting in a point on z axis high above, how would our cone look like?

The students were left to visualise and think about it for a while. In both groups approximately 10% of them answered the question correctly (the correct answer was that it would look like a circle with a point in the middle), whereas the rest did not know the answer. After this they were asked to use the "rotate" tool in order to see in GeoGebra what was the correct answer.

2. If we were sitting in point (3,0,3) which shape would our cone assume?

Again the same approach as with the first question has been applied. No students could give an answer, correct or not (correct answer was that it would look like an isosceles triangle).

7.3 Second week

During the lesson in second week the students worked with wooden objects. Each student was given one wooden object to work with. They were first told to take measurements of the object and sketch the objects on paper, after what they had to translate the measurements into Cartesian coordinates and sketch the object in GeoGebra.

With regard to the first group subjects 10, 9, 11, 4, 6 and 23 (27%) completed the task two times faster than the rest of the subjects and they had no problems with it whatsoever. Other subjects had problems with translating the measurements of the wooden objects into

Cartesian coordinates. The reason for this could be either not quality mathematics pre-knowledge or problems with visualisation.

Second group was much quicker in understanding and applying the learned content, what could be explained with the fact that the second group had a break in between lectures before the lesson whereas the first group had no breaks and arrived to the class straight from another one. Subjects 29, 44, 46, 41, 36, 28 and 29 (32%) have completed the task two times faster than the rest of the subjects. Subjects 50 and 35 completed the task three times faster and more effective than the rest of the subjects and displayed clear problem formulation, quick translation of measurements into Cartesian coordinates as well as dexterous use of GeoGebra for sketching their wooden object.

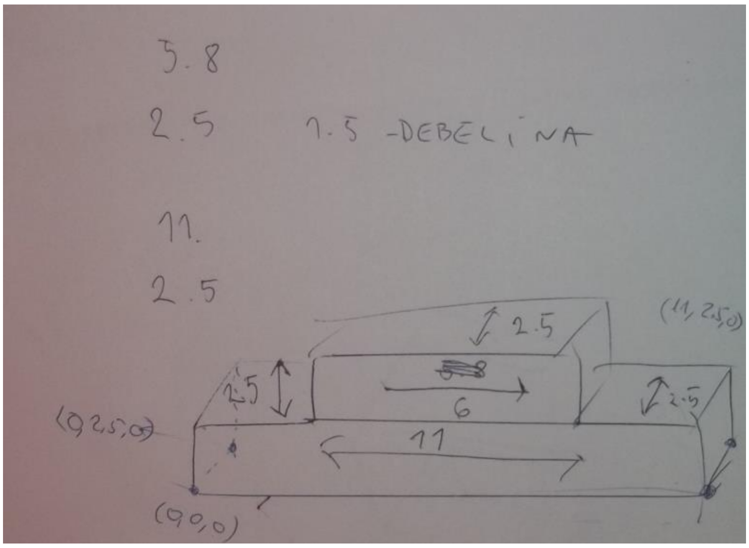

Figure 13. Student's sketch of object 16

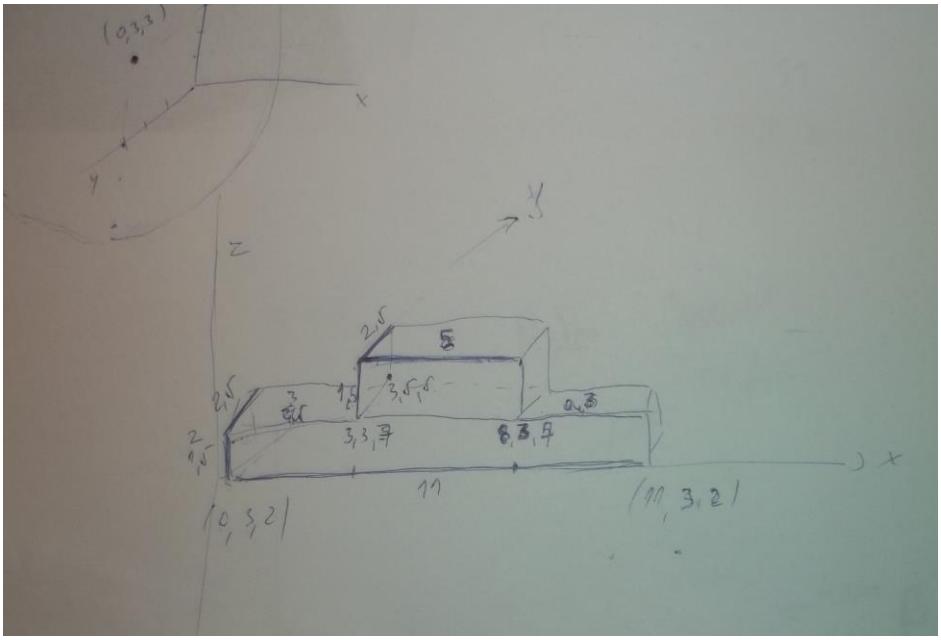

Figure 14. Another student's sketch of object 16

Figure 15. Object 16

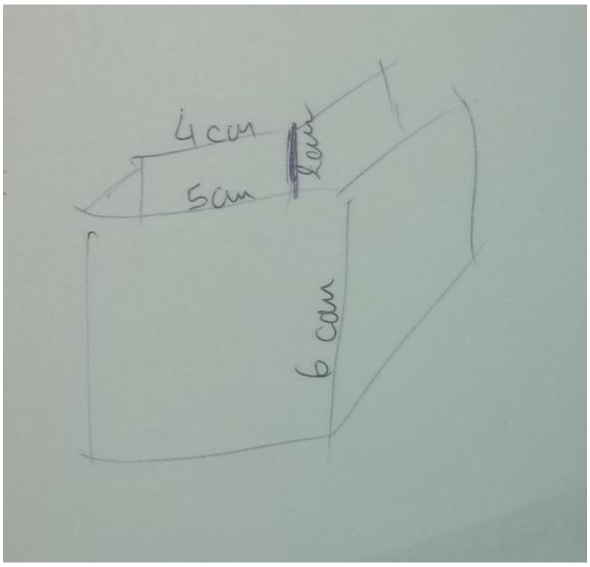

Figure 16. Student's sketch of object 18

Figure 17. Object 18

After sketching wooden objects in GeoGebra, orthogonal sketches of wooden objects the students have not worked with before were introduced, containing conventionally front, side and top view. The students' task was to sketch the objects in GeoGebra, using the orthogonal sketches. A short explanation was given on what the orthogonal sketches contain and definitions of each view, after what it was proceeded to the task.

Students' performance on this task has been significantly faster and more effective than their performance on the previous task, the reasons for which could be: 1) they gained practice and experience during translating measurements of objects they were to sketch in GeoGebra into Cartesian coordinates, 2) sketching in GeoGebra and rotating their sketch of the object helped with visualisation process which was required for this task as well. One mitigating circumstance in this case might have been that the objects were all conceptually similar so practice during the first task might have made the second task substantially easier for the subjects.

It has to be noted that the students predominantly used the isometric projection as their reference guide, which, given the fact that they have little or no previous experience (due to their secondary education) with orthogonal sketches and only used the front, side and top view for measurements. Additionally, even those subjects who had difficulties with the previous task exhibited more effective task solving in this part of the lesson. Due to the lack of time however, some of the subjects did not succeed in completing this task, although the greater majority (81.81%) has. Those who have completed their task were given the wooden objects for examination which they have been sketching for the purpose of comparison.

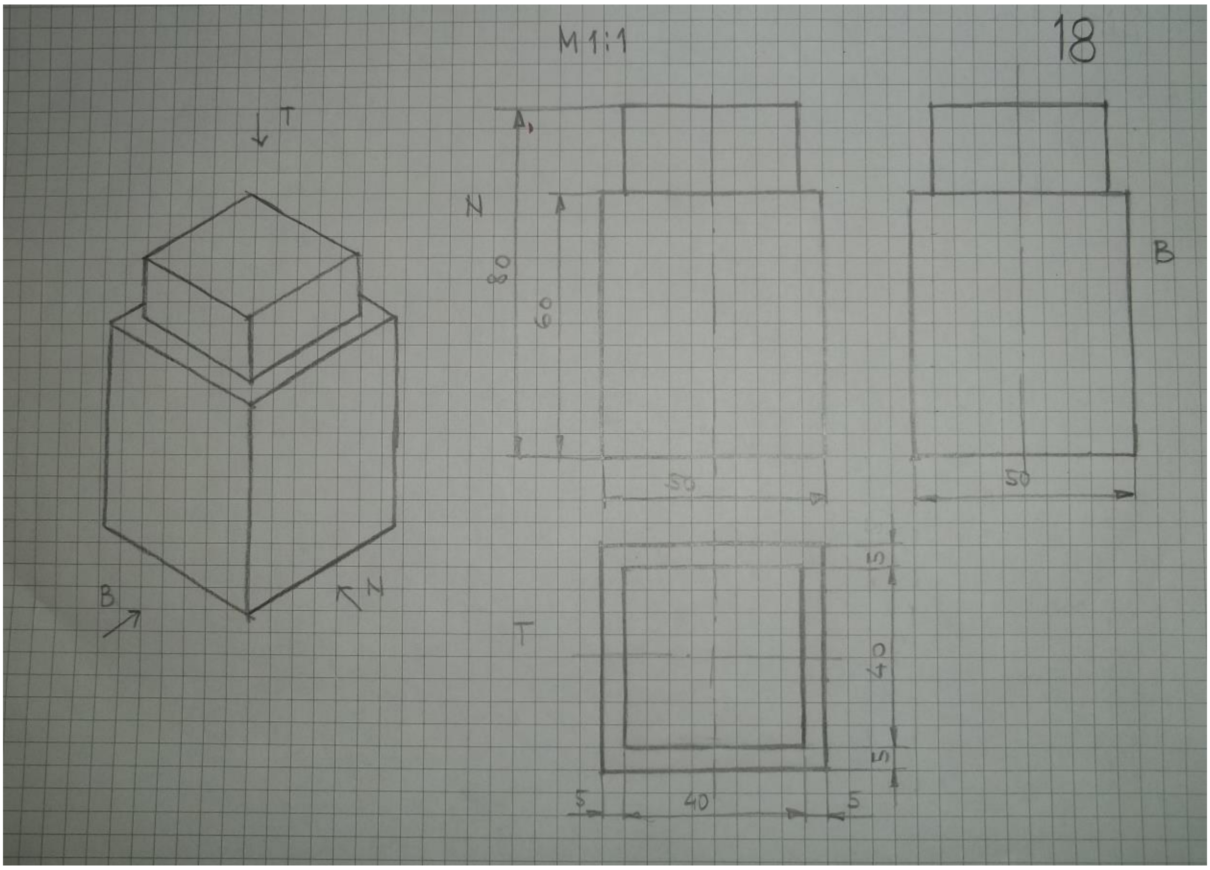

Figure 19. Orthogonal sketch of object 18

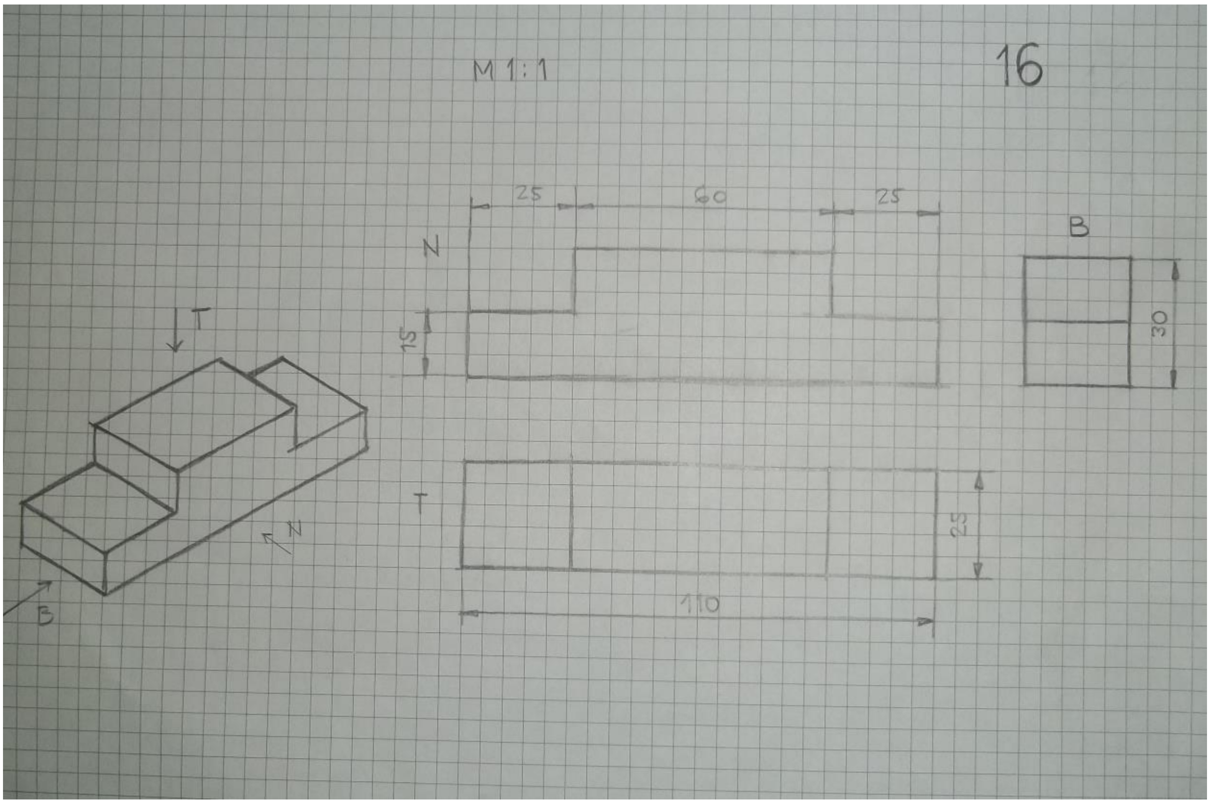

Figure 18. Orthogonal sketch of object 16

7.4 Third week

Exercises given in third week were that of rotation and reflection in 2D. GeoGebra apps have been made and uploaded on the GeoGebraTube, which students could easily access and then work with them. All in all there were three exercises for reflection and three for rotation.

Exercise 1.

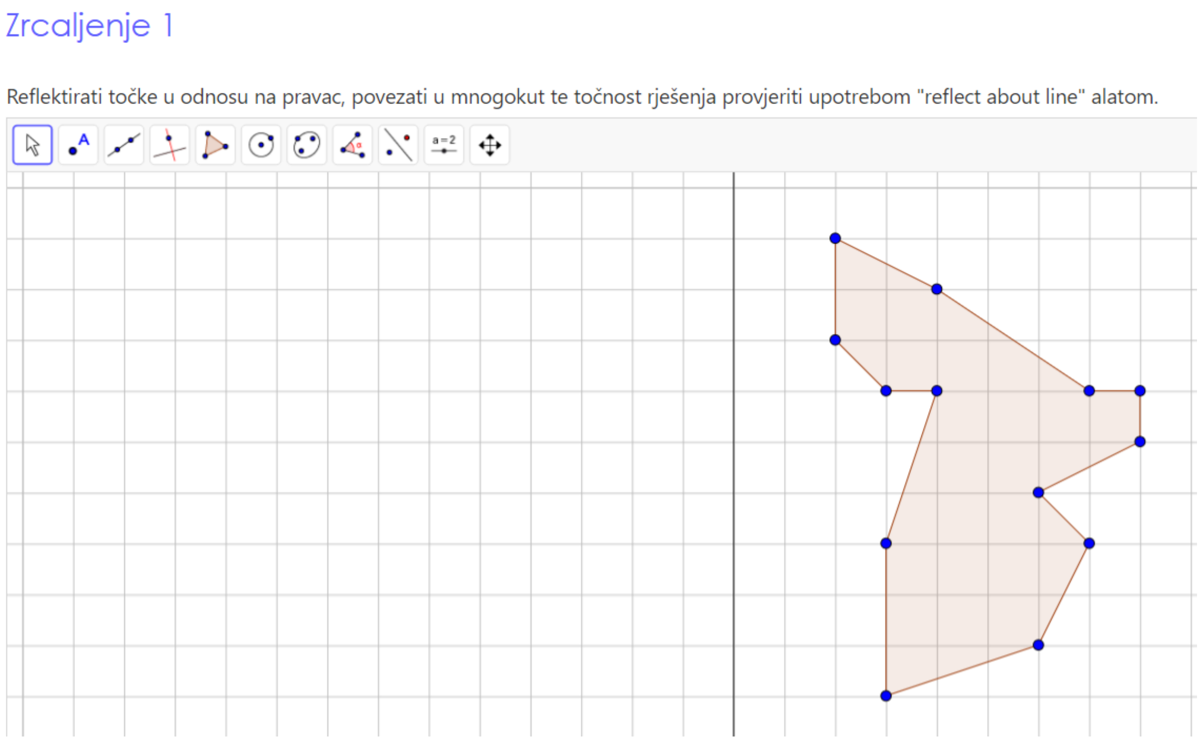

Figure 20. Reflection exercise 2

In this exercise it was required of the subjects to reflect all points with respect to the line and connect them into a polygon. Correct solution could be tested by using the 'reflect about line' tool. This exercise was intended to be an introductory, easy exercise for other two, the level of which was designed according to the scaffolding rule.

Exercise 2.

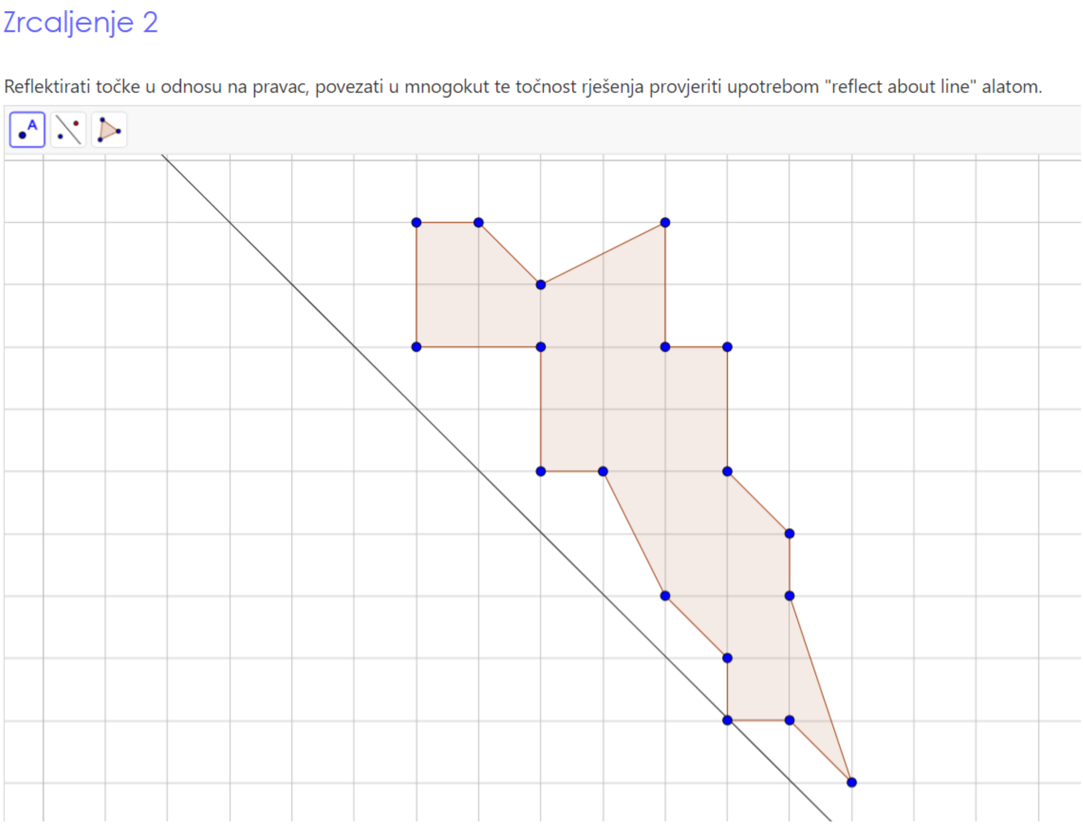

Figure 21. Reflection exercise 2

The subjects were required to once again reflect the points with respect to the line which was in this exercise sketched under a certain angle, which made the exercise a little more difficult than the previous one.

Exercise 3.

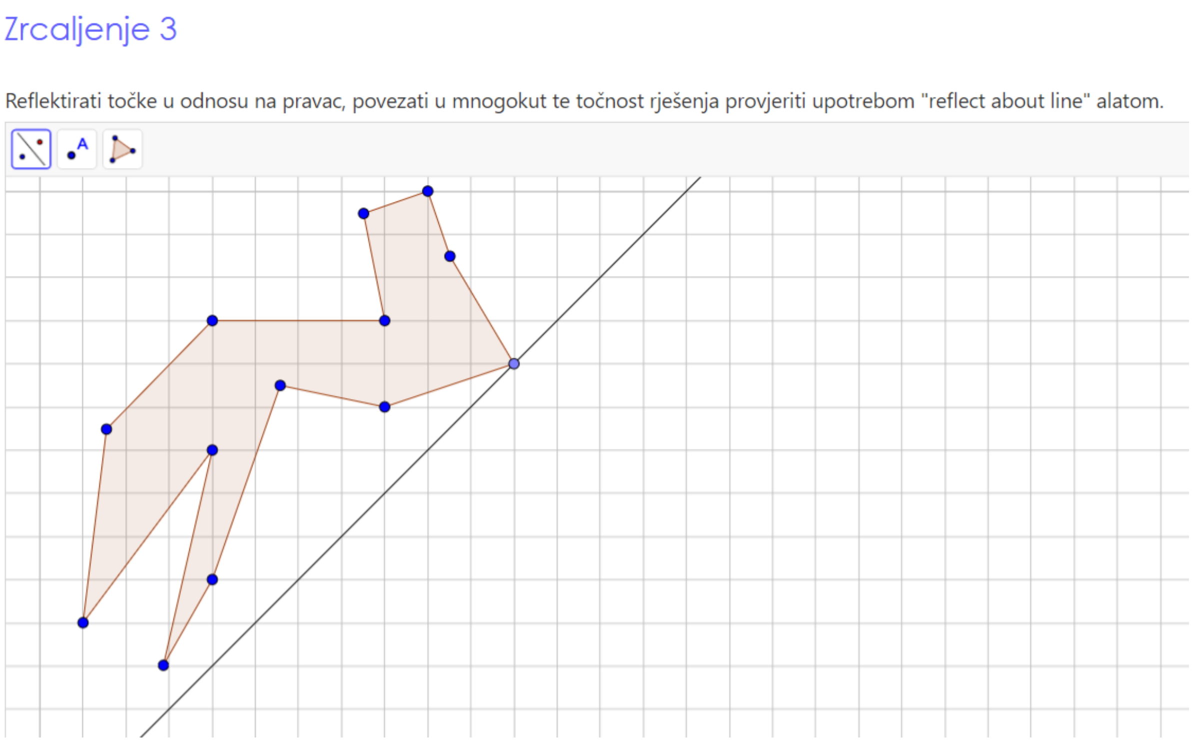

Figure 22. Reflection exercise 3

The line in this exercise has once again been drawn under a specific angle but the additional difficulty of this exercises lied in the fact that some of the points have been drawn in the middle of the grid squares.

Exercise 4.

Rotacija 1

Zarotirati nacrtan lik oko točke G u smjeru kazaljke na satu za 90 stupnjeva. Provjeriti točnost rješenja uz pomoć alata "rotate around point".

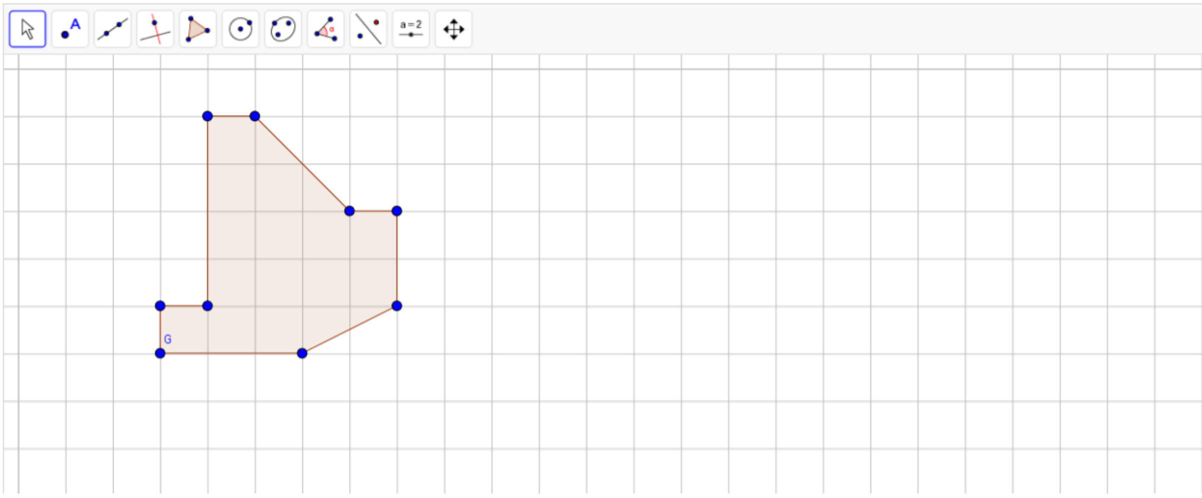

Figure 23. Rotation exercise 1

In this exercise the polygon needed to be rotated 90° clockwise around point G, which was the only point labelled. The correct solution could be checked using GeoGebra’s ‘rotate around point’ tool. This exercise has been designed once again to serve as an introductory exercise, with the other two following in difficulty level according to the scaffolding rule.

Exercise 5.

Rotacija 2

Zarotirati nacrtan lik oko točke A u smjeru kazaljke na satu za 90 stupnjeva. Provjeriti točnost rješenja uz pomoć alata "rotate around point".

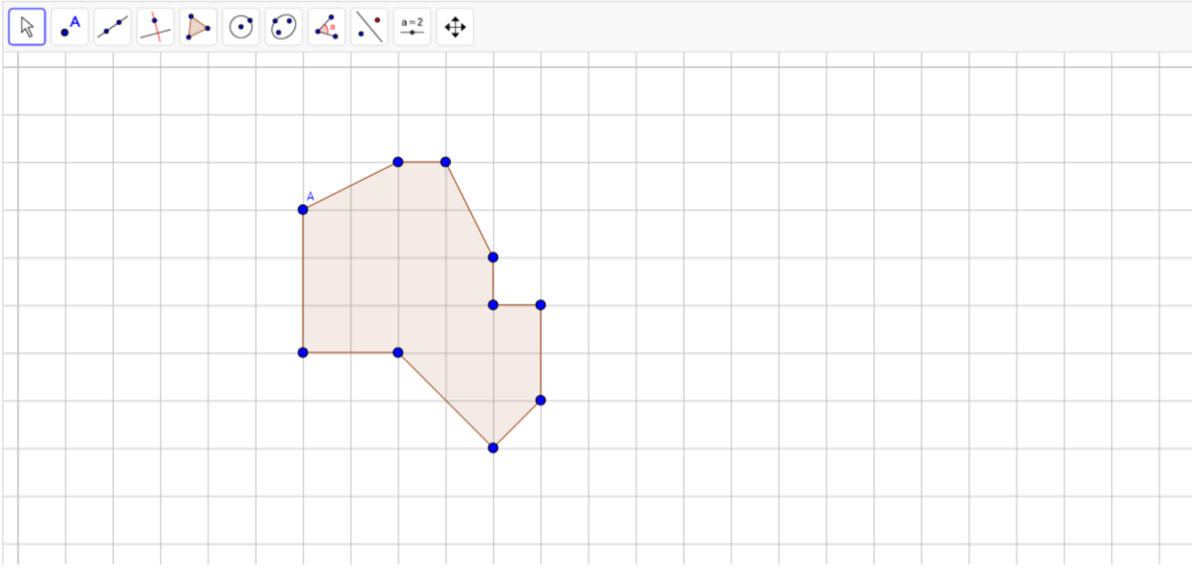

Figure 24. Rotation exercise 2

In this exercise the polygon needed to be rotated 90° clockwise around point A, which was the only point labelled. The difference in difficulty with regard to the previous exercise can be noted in the increased number of oblique lines, as well as the position of the point around which the polygon needed to be rotated.

Exercise 6.

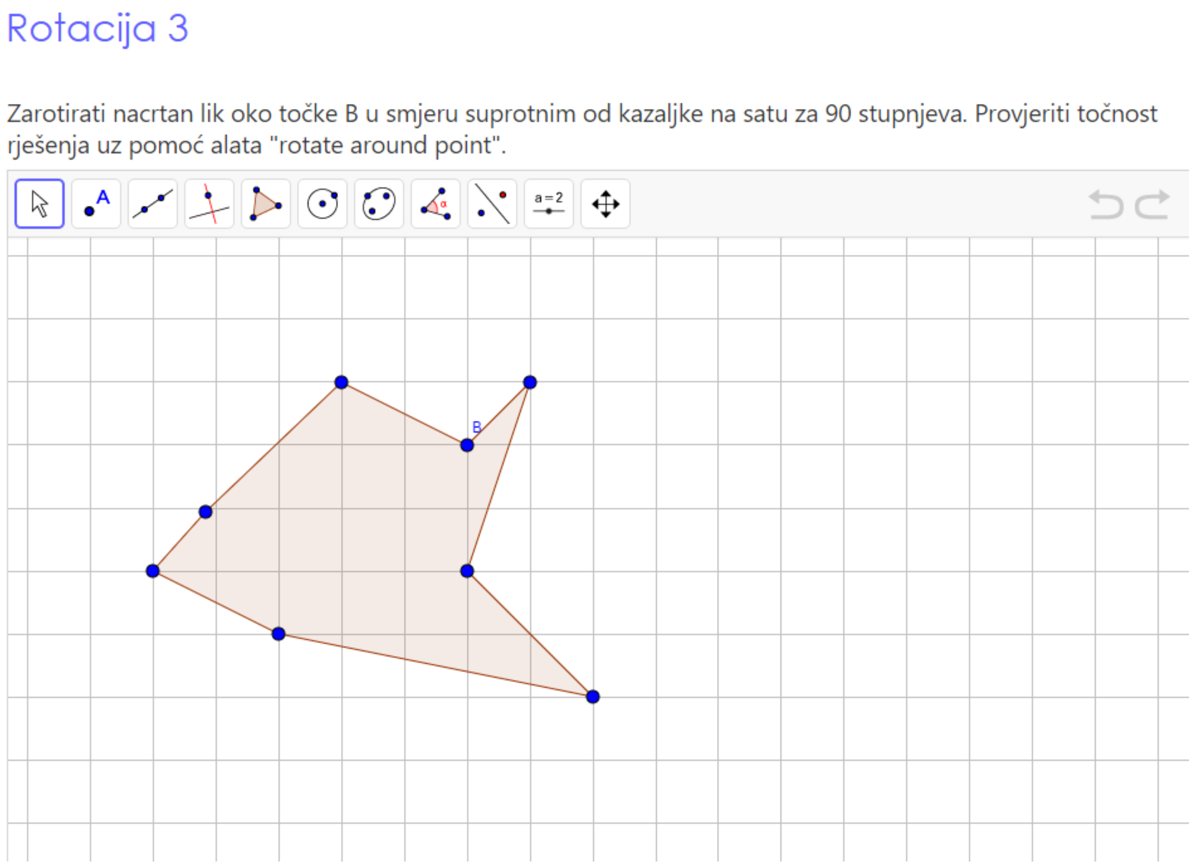

Figure 25. Rotation exercise 3

The given polygon needed to be rotated 90° anticlockwise around point B. There are no straight lines in this exercise which is why it has been given to the students after completing the first two rotation exercises.

Certain subjects have completed all tasks within thirty minutes whereas some found it difficult to complete them within an hour and thirty minutes. Certain subjects encountered technical difficulties what made the task solving run slower. Again there was a noticeable difference between first and second group, with the second group completing tasks within averagely 25 minutes, which could once again be contributed to the fact that the second group had a break between the lessons.

In the first group, subject 19 completed the task as first in the group, whereas subjects 6, 10, 11, 2, 23 and 27 solved the tasks faster than the rest of the subjects (27%).

In the second group, subject 31 completed the task as first in the group. Subject 51 also stood out with dexterity and speed of task solving. Subject 37 excelled in the whole group by solving all six tasks twice within 30 minutes.

7.5 Fourth week

For classes in last week mathematics problems with emphasis on visualisation have been designed, again following the four step method. However, the exercises have been divided into two parts, level one (basic) and level two (advanced).

Exercise 1. **Level one**: Base length of a right regular square pyramid is 4, whereas the height of the pyramid is 8. The base of the pyramid cuts a sphere the diameter of which is 4 into two equal parts. Sketch the pyramid and the sphere.

Level two: Sketch the part of the pyramid which is not a part of the sphere.

Step 1/2: The students were required to first attempt to visualize the given problem and then sketch it on a piece of paper. At this point only subject 2 from group one (4,5%) and subjects 50 and 49 from group two (9%) managed to complete Level two. Subject 50 also pointed out that the shape looked like the Eiffel tower, which was the correct answer but what also showed that he was able to compare the resulting shape with others in his mind.

Step 3: Sketching in GeoGebra. Given that they already had plenty of practice with working in this software, there were no problems with this except technical ones.

Step 4: Since Level two task had to be given at the beginning of the exercise in order to avoid the students already seeing the answer on the sketch in GeoGebra, in Step 4 it was only left to the students to compare their sketches on paper with those in GeoGebra.

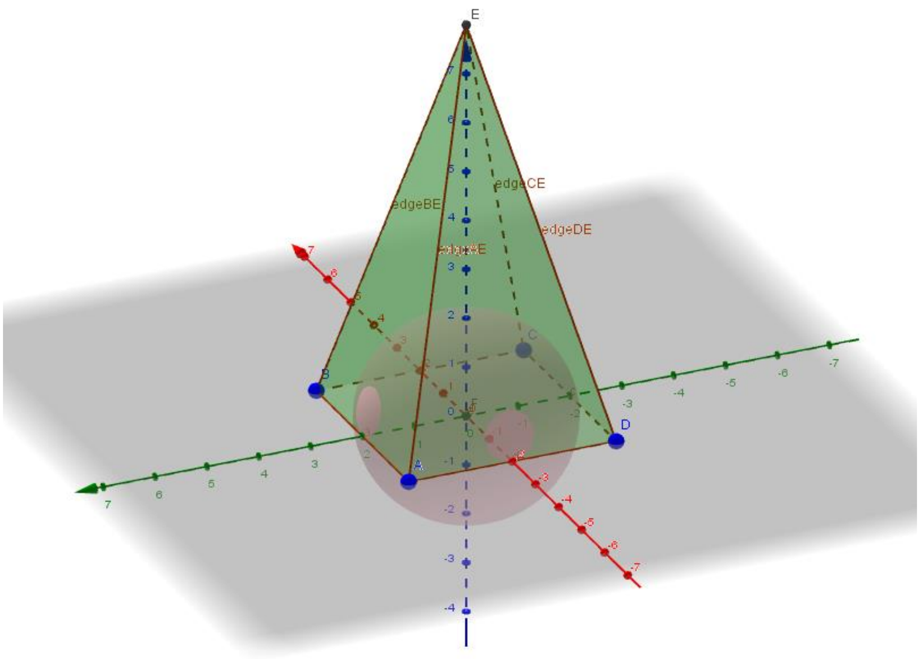

Figure 26. Correct sketch of Exercise 1 in GeoGebra

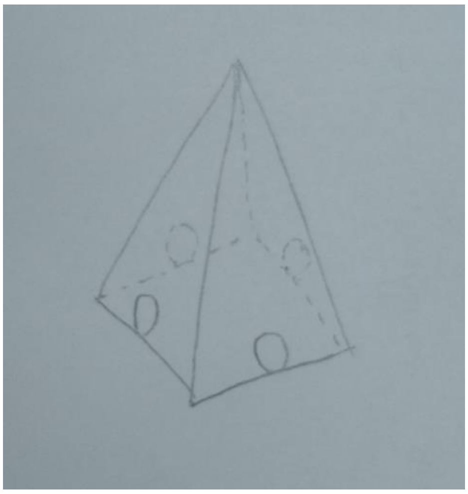

Figure 27. Correct solution of Level two in Exercise 1

<u>Exercise 2.</u> Level one: One face of a cube, the length of the edge of which is 2, lies on the base of a right circular cylinder, the diameter of which is 4, whereas the height of the cylinder is 4. Sketch the cube and the cylinder.

Level two: A plane runs along the bisector of one of the cube edges. Sketch how the intersection of the plane, cylinder and cube looks like.

Step 1/2: The students were required to first attempt to visualise the given problem and then sketch it on a piece of paper. At this point no one managed to complete Level two.

Step 3: Sketching in GeoGebra.

Step 4: Since Level two task had to be given at the beginning of the exercise in order to avoid the students already seeing the answer on the sketch in GeoGebra, in Step 4 it was only left to the students to compare their sketches on paper with those in GeoGebra. Additionally, since no one has managed to complete Level two, the subjects have been encouraged to take their time inspecting their GeoGebra sketches to see what would have been the correct answer.

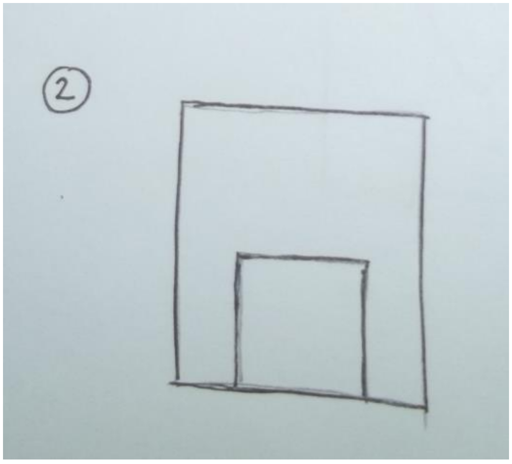

Figure 28. Correct solution of Level two in Exercise 2

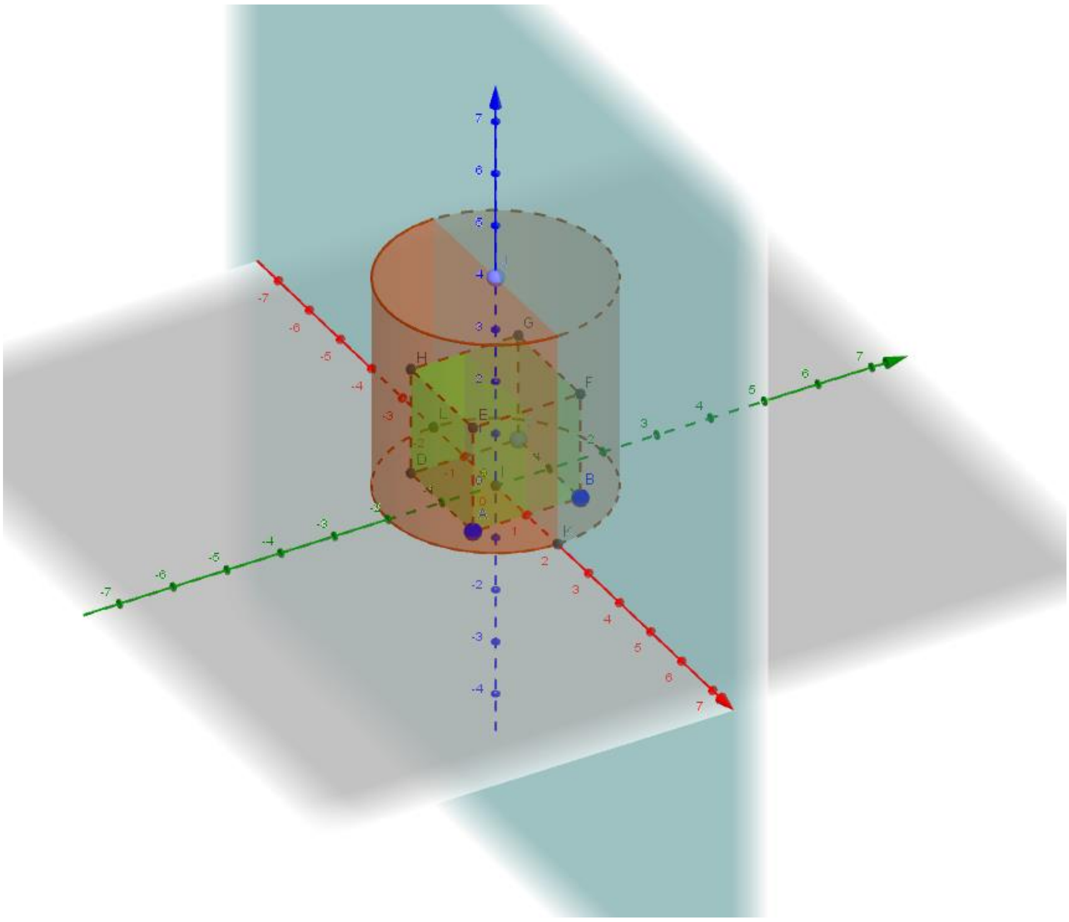

Figure 29. Correct sketch of Exercise 2 in GeoGebra

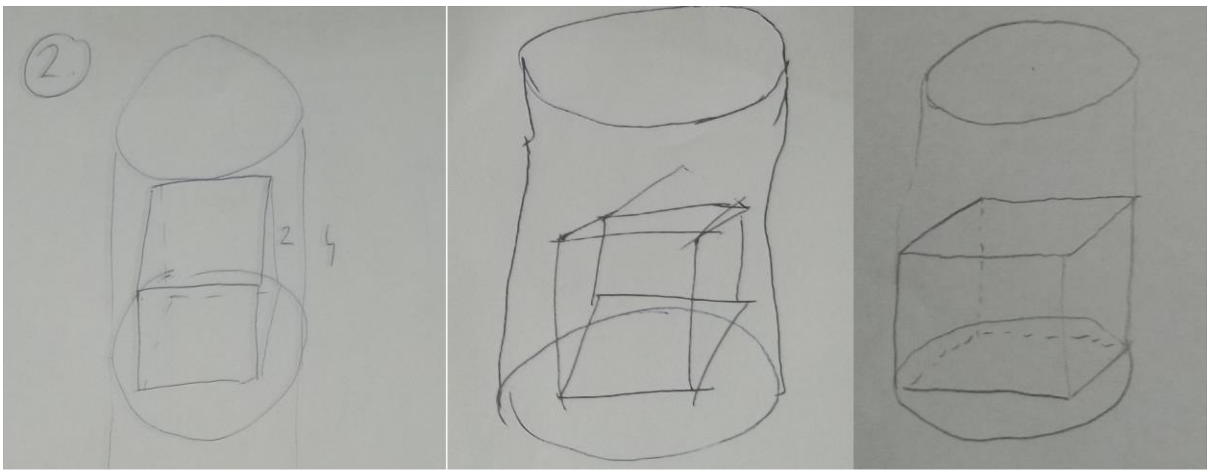

Figure 30. Sketches on paper of Exercise 2 by three different subjects

<u>Exercise 3.</u> **Level one**: Diameter of the base of a right circular cylinder is 4, whereas the height of the cylinder is 4 as well. On top of the cylinder lies a right regular square pyramid, the length of the edge of the base of which is 2, whereas its height is 3. Sketch the cylinder and the pyramid.

Level two: A plane runs along the bisector of one of the edges of the pyramid base. Sketch how the intersection of the plane, pyramid and cylinder looks like.

<u>Step 1/2</u>: The students were required to first attempt to visualise the given problem and then sketch it on a piece of paper. Again no one managed to complete Level two.

<u>Step 3</u>: Sketching in GeoGebra.

<u>Step 4</u>: Since Level two task had to be given at the beginning of the exercise in order to avoid the students already seeing the answer on the sketch in GeoGebra, in Step 4 it was only left to the students to compare their sketches on paper with those in GeoGebra. Additionally, since no one has managed to complete Level two, the subjects have been encouraged to take their time inspecting their GeoGebra sketches to see what would have been the correct answer.

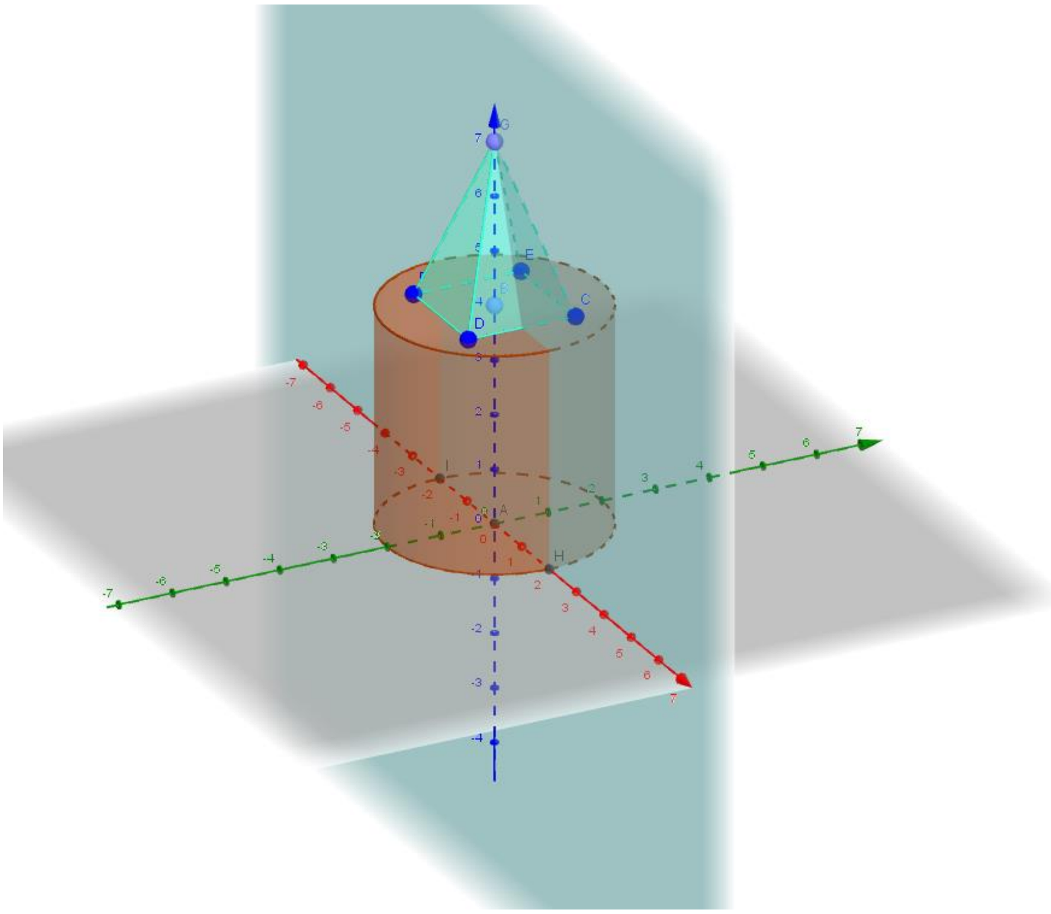

Figure 31. Correct sketch of Exercise 3 in GeoGebra

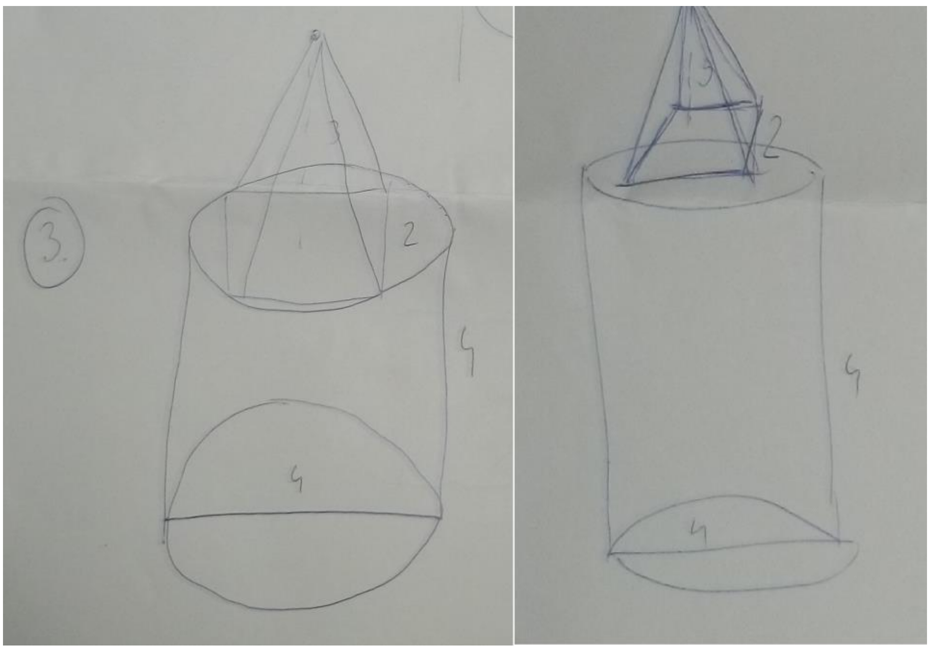

Figure 32.Sketches on paper of Exercise 3 by two different subjects

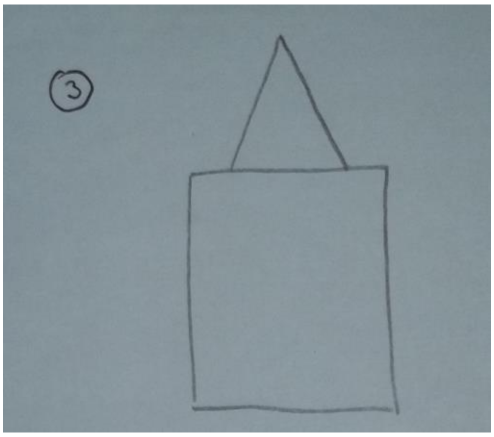

Figure 33. Correct solution of Level two in Exercise 3

Exercise 4. **Level one:** Length of one of the edges of the cube is 4. A sphere lies within the cube, the diameter of which is also 4. Sketch the cube and the sphere.

Level two: A plane runs along the diagonal of one of the cube faces. Sketch how the intersection of the plane, cube and the sphere looks like.

Step 1/2: The students were required to first attempt to visualise the given problem and then sketch it on a piece of paper. Subjects 19, 4 and 2 from group one (13%) and subjects 35 and 50 from group two (9%) managed to complete Level two. The reason for this could be that thanks to the previous exercises and rotating their GeoGebra sketches the visualisation process was easier for some.

Step 3: Sketching in GeoGebra.

Step 4: Since Level two task had to be given at the beginning of the exercise in order to avoid the students already seeing the answer on the sketch in GeoGebra, in Step 4 it was only left to the students to compare their sketches on paper with those in GeoGebra.

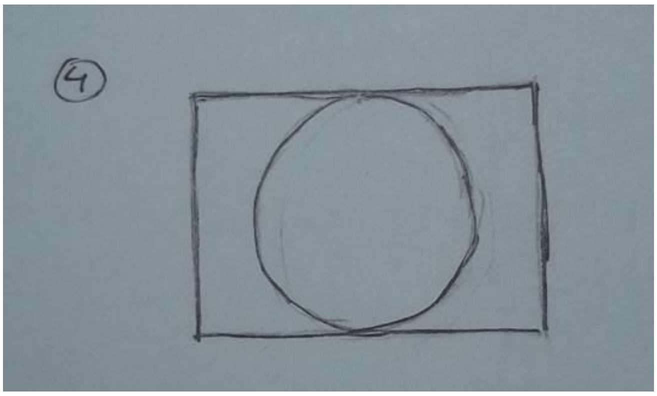

Figure 34. Correct sketch of Level two in Exercise 4

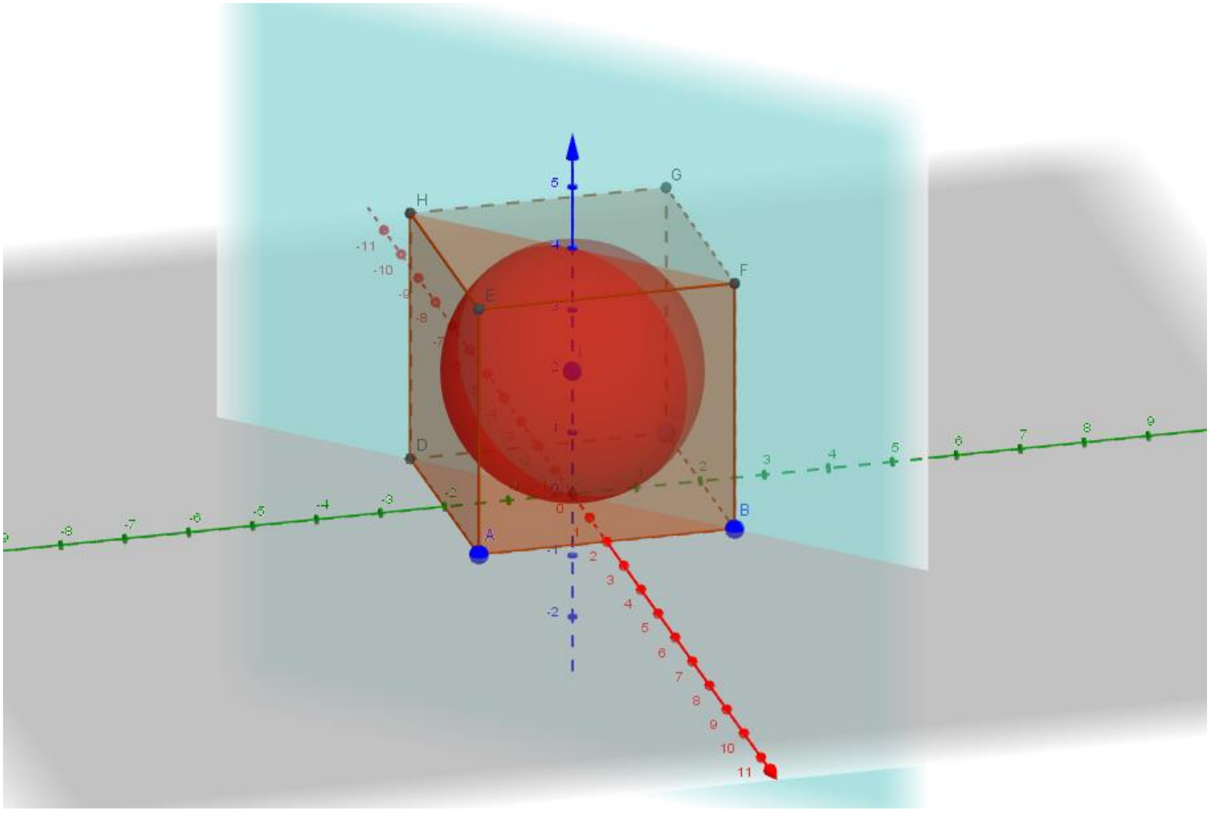

Figure 35. Correct sketch of Exercise 4 in GeoGebra

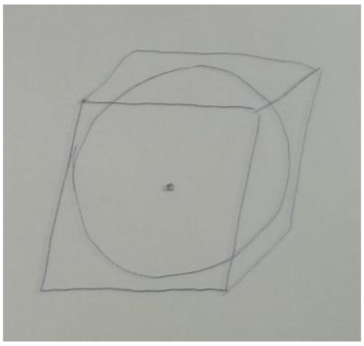

Figure 36. Sketch of Exericse 4 by one of the subjects

Last exercise was not completed due to the lack of time, but it will be outlined in the following section.

Exercise 5. **Level one**: Length of the edge of a cube is 4. On one of the faces of the cube lies the base of a right regular cone. Base diameter of the cone is 4 whereas the height of the cone is 2. The cone however lies within the cube. Sketch the cube and the cone.

Level two: A plane runs along the diagonal of one of the cube faces. Sketch how the intersection of the plane, cube and the cone looks like.

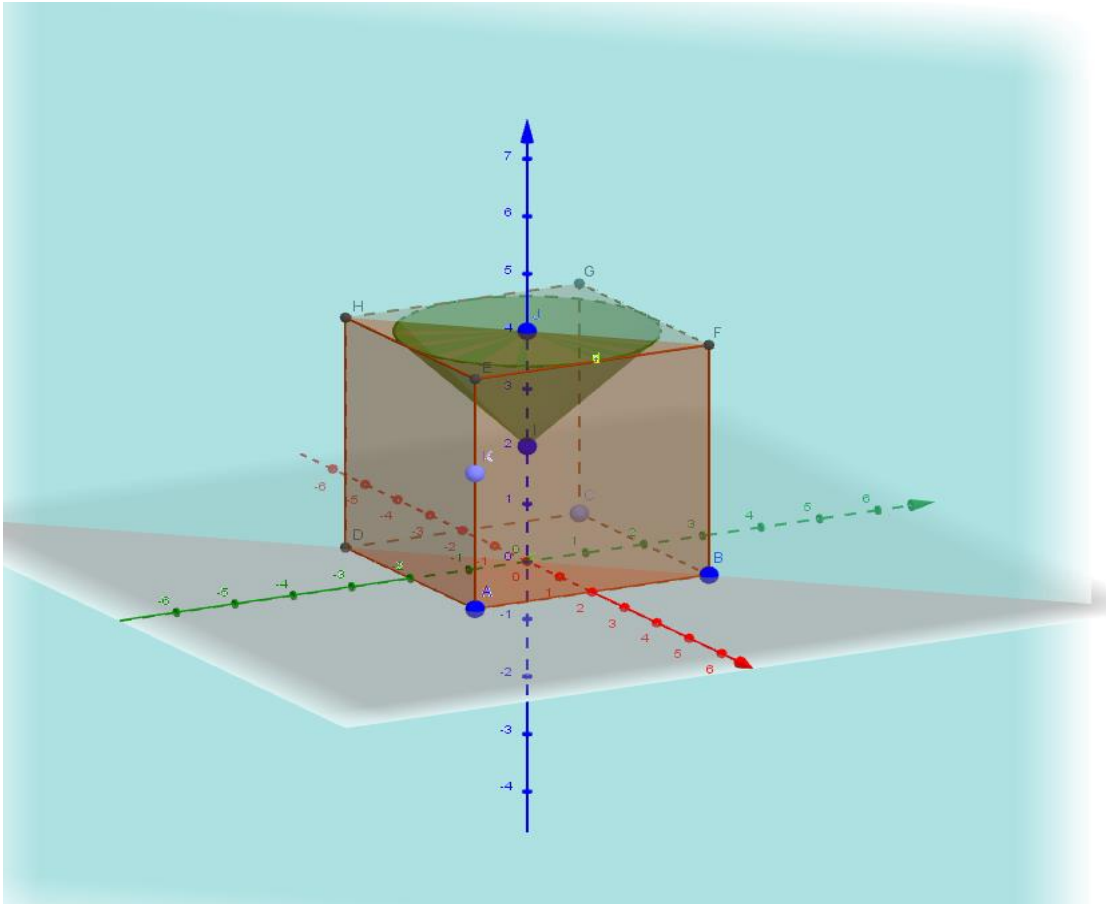

Figure 37. Correct sketch of Exercise 5 in GeoGebra

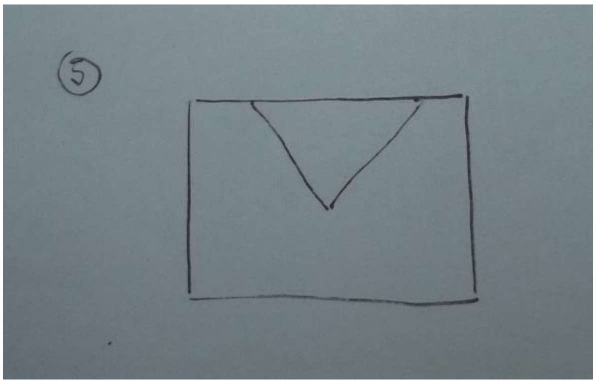

Figure 38. Correct sketch of Level two in Exercise 5

8 Results and discussion

8.1 Employed instruments and tools

For pre-test the Spatial test by Pauline Smith and Chris Whetton (1988) has been used. After the completed experimental programme the test has been applied once again, along with a questionnaire containing 22 items dealing with possible predictors of highly developed spatial skills, using Lickert scale. For the greater part the questionnaire contains items from the questionnaire written by Dr. Terry Armstrong (http://literacyworks.org/mi/assessment/findyourstrengths.html) whereas some of the items have been added, which were tested as predictors for highly developed spatial abilities by other researchers. Specifically, Sorby and Baartmans (2000) have conducted a research in 1993 on the sample of first-year students (418 male and 117 female, 535 in total) who enrolled mechanical, civil, environmental, geological, and general engineering, administering PSVT:R along with a background questionnaire. From the analyses of the results on the PSVT:R and the answers on the background questionnaire the authors have isolated four significant predictors of success on the PSVT:R: 1) play as children with construction toys, 2) gender, 3) math scores, 4) previous experience in design-related courses like drafting, mechanical drawing etc. (Sorby & Baartmans, 2000). It was for this reason that it has been decided to include items in the questionnaire which examined these possible predictors. Additionally Spatial ability test by Paul Newton and Helen Bristoll (2009) has been administered.

This research has used GeoGebra as the primary spatial skills training tool with the aim of verifying the value of this software for the purpose of spatial skills training. GeoGebra is one of the few mathematical software editions which have been translated into Croatian and Šuljić (2005) found that GeoGebra has grown in popularity in the Republic of Croatia because, among other reasons, it is freeware. As other reasons for this popularity he gives: 1) it is a professionally made program, which has won many European software rewards (including those for educational software); 2) it has been translated to Croatian; 3) it covers mathematical programs for elementary and secondary schools in the Republic of Croatia well; 4) is able to bring geometry and algebra closer than any other program; 5) entails an intuitive algebraic equation input (e.g. $(x-3)^2 + (y+2)^2 = 25$); 6) it is easy to use for both teachers and students; 7) a student can use it since the fifth grade of elementary school until he graduates from secondary school; 8) it has high-quality graphics, suitable especially for classroom projections; 9) can easily produce a dynamic drawing on a web page (applet); 10) the constructions can be transferred to other presentations or programs, including LaTEX (Šuljić, 2005).

8.2 Sample

The experimental group consisted of 35 males (67.3%) and 17 females (32.7%) whereas the control group consisted of 33 males (63.5%) and 19 females (36.5%), all studying at the Faculty of Science and Education in Mostar, Bosnia and Herzegovina, only with focus on different subjects. The experimental group took the experimental programme whereas the control group has not been influenced in any way. Pre-test and post-tests were taken by both experimental and control group at the approximately the same time.

Group	Gender	Frequency	Percent
Experimental	M	35	67.3
	F	17	32.7
	Total	52	100.0
Control	M	33	63.5
	F	19	36.5
	Total	52	100.0

Table 6. Research sample

8.3 Variables

Dependent variables:

- Students' success during the experimental programme
- Students' results on the spatial abilities pre-test and post-test

Independent variables:

- Students' gender

8.4 Hypotheses

General hypothesis: it is possible to enhance spatial abilities via specially developed programme to use in Geogebra throughout the span of minimum one month. According to previous research enhanced spatial abilities will serve as a foundation for the students for success in their studies and also in their work in the future.

1. Differences between the control and experimental group at initial testing of spatial abilities are not statistically significant with regard to any subsample.
2. Differences between the control and experimental group at final testing of spatial abilities are statistically significant.
3. Differences between initial and final testing of spatial abilities within the control group are not statistically significant.
4. Differences between initial and final testing of spatial abilities within the experimental group are statistically significant.

Auxiliary hypotheses:

1. There are no statistically significant differences between the genders neither with regard to the experimental programme (initial and final testing) nor with regard to the control group.
2. Knowledge gained during high school education does not have a statistically significant impact on the differences between the students' success within the experimental group (continued monitoring of advancement during the experimental programme as well as initial and final testing).

8.5 Results

Test	Group	N	M	SD	min	max
Initial Smith & Whetton	Experimental	49	43.20	14.74	12.00	69.00
	Control	44	52.68	8.53	36.00	71.00
	Total	93	47.69	13.04	12.00	71.00
Final Smith & Whetton	Experimental	40	55.93	11.42	22.00	76.00
	Control	44	53.36	9.89	37.00	73.00
	Total	84	54.58	10.66	22.00	76.00
Final Newton & Bristoll	Experimental	41	30.54	5.07	18.00	40.00
	Control	45	31.00	5.89	13.00	43.00
	Total	86	30.78	5.49	13.00	43.00

Table 7. Subjects' mean scores on tests

On the pre-test by Smith and Whetton (1988) the experimental group scored a mean of 43.20 (with worst score being 12 and best score 69) and standard deviation of 14.74 whereas the control group scored a mean score of 52.68 (with worst score being 36 and best score being 71) and standard deviation of 8.53. A noticeable larger standard deviation for the experimental group scores on the pre-test should be pointed out.

On the Smith and Whetton (1988) post-test the experimental group scored a mean of 55.93 (with worst score being 22 and best score 76) and standard deviation of 11.42 whereas the control group scored a mean score of 53.36 (with worst score being 37 and best score being 73) and standard deviation of 9.89. A less pronounced differences in the standard deviations for both groups when compared to each other can be observed.

On the whole a difference between the mean score on the Smith and Whetton (1988) spatial abilities test for the experimental group with regard to average results can be observed: 43.20 on the pre-test and 55.93 on the post-test, whereas the difference between the average scores of the control group is less pronounced: 52.68 on the pre-test and 53.36 on the post-test.

On the Newton and Bristoll (2009) spatial abilities test administered as post-test the experimental group scored a mean of 30.54 (with 18 out of maximum 45 as worst score and 40 as best score) with standard deviation 5.07. On the same test the control group scored an average result of 31 (with 13 out of maximum 45 as worst score and 43 as best score) with standard deviation 5.89.

Item	Group	Total		
		N	M	SD
1. When I was a child I enjoyed playing games which included putting different pieces together (Lego blocks, jigsaw puzzles, 3D models etc).	Experimental	41	3.68	1.128
	Control	45	3.42	1.323
	Total	86	3.55	1.233
2. When I was a child I enjoyed solving Sudoku, labyrinths and 'connect the dots' problems.	Experimental	41	3.05	1.224
	Control	45	3.20	1.375
	Total	86	3.13	1.300
3. When I was a child I loved to play chess.	Experimental	41	2.34	1.543
	Control	45	2.91	1.411
	Total	86	2.64	1.494
4. I was good at science subjects at school.	Experimental	41	3.34	1.039
	Control	45	3.76	1.069
	Total	86	3.56	1.069
5. I was in scouts when I was a child.	Experimental	41	1.59	1.204
	Control	45	1.62	1.248
	Total	86	1.60	1.220
6. When I was a child I spent a lot of time outdoors.	Experimental	41	4.37	.968
	Control	45	4.22	.974
	Total	86	4.29	.969
7. When I am walking around unfamiliar city I can easily retrace my steps back to the starting point.	Experimental	41	3.44	1.074
	Control	45	3.73	1.250
	Total	86	3.59	1.172
8. I have no problems parking a car (in case you are not driving, please leave empty).	Experimental	41	3.85	1.085
	Control	42	4.29	1.019
	Total	83	4.07	1.068
9. Use of numbers and symbols is easy for me.	Experimental	41	3.63	.968
	Control	45	4.16	1.186
	Total	86	3.91	1.113
10. Music is very important to me in everyday life.	Experimental	41	4.05	1.117
	Control	45	4.29	1.036
	Total	86	4.17	1.076
11. I always know where I am with regard to where I live.	Experimental	41	4.02	1.084
	Control	45	4.29	1.141
	Total	86	4.16	1.115
12. I often develop equations in order to describe correspondences or explain what I see.	Experimental	41	2.12	1.187
	Control	45	2.64	1.433
	Total	86	2.40	1.340

13. I do not get lost easily and the use of maps and landmarks is easy for me.	Experimental	41	3.49	1.121
	Control	45	3.67	1.261
	Total	86	3.58	1.193
14. I have a very good sense of pitch, tempo and rhythm.	Experimental	41	3.29	1.289
	Control	45	3.33	1.187
	Total	86	3.31	1.230
15. Knowing directions is easy for me.	Experimental	41	3.78	1.107
	Control	45	3.96	1.086
	Total	86	3.87	1.093
16. I have good balance and eye-hand coordination and enjoy sports which use a ball.	Experimental	41	3.76	1.179
	Control	45	4.09	1.145
	Total	86	3.93	1.166
17. I have the ability to represent what I see by drawing or painting.	Experimental	41	2.41	1.183
	Control	44	2.84	1.238
	Total	85	2.64	1.223
18. My excellent coordination and balance enable me to be very successful in activities which demand high speed.	Experimental	41	3.49	1.186
	Control	44	3.61	1.243
	Total	85	3.55	1.210
19. I like to think about numerical issues and examine statistics.	Experimental	41	2.32	1.331
	Control	45	2.82	1.403
	Total	86	2.58	1.384
20. My ability to draw is recognised and complimented by others.	Experimental	41	1.83	1.116
	Control	45	2.18	1.267
	Total	86	2.01	1.203
21. I can remember the tune of a song when asked.	Experimental	41	2.78	1.406
	Control	45	3.07	1.355
	Total	86	2.93	1.379
22. It is easy for me to visualise things in 3D.	Experimental	41	3.41	1.048
	Control	45	3.67	1.066
	Total	86	3.55	1.059

Table 8. Subjects' answers on individual items in the questionnaire

The aim of this questionnaire was to determine whether there was a statistically significant correlation between specific items in the questionnaire and high results on the initial spatial abilities test. The questionnaire itself employed Lickert type scale. On the whole the subjects both in experimental and in control group answered the lowest average of 1.59 (SD=1.204) for experimental and 1.62 (SD=1.248) for control group with total average of 1.60 (SD=1.220) for item number 5, “I was in scouts when I was a child”. Second item with lowest average answers was item number 20, “My ability to draw is recognised and complimented by others”: average of 1.83 (SD=1.116) for experimental and 2.18 (SD=1.267) for the control group, with the total average of 2.01 (SD=1.203). Other observable items on which average answers were relatively low were:

1. Item number 21: “I can remember the tune of a song when asked”.

Experimental group 2.78 (SD=1.406), control group 3.07 (SD=1.355) and total average 2.93 (SD=1.379)

2. Item number 3: “When I was a child I loved to play chess”.

Experimental group 2.34 (SD=1.543), control group 2.91 (SD=1.411) and total average 2.64 (SD=1.494)

3. Item number 17: “I have the ability to represent what I see by drawing or painting”.

Experimental group 2.41 (SD=1.183), control group 2.84 (SD=1.238) and total average 2.64 (SD=1.223)

4. Item number 19: “I like to think about numerical issues and examine statistics”.

Experimental group 2.32 (SD=1.331), control group 2.82 (SD=1.403) and total average 2.58 (SD=1.384)

5. Item number 12: “I often develop equations in order to describe correspondences or explain what I see”.

Experimental group 2.12 (SD=1.187), control group 2.64 (SD=1.433) and total average 2.4 (SD=1.34)

With regard to high average answers, the highest average answer was for item number 6 (“When I was a child I spent a lot of time outdoors”) with average answer of 4.37 (SD=0.968) for experimental, 4.22 (SD=0.974) for control group and total average answer 4.29 (SD=0.969). Second highest average answer was for item number 10 (“Music is very important to me in everyday life”) with average answer of 4.05 (SD=1.117) for experimental, 4.29 (SD=1.036) for control and total average answer 4.17 (SD=1.076).

Other observable items on which average answers were relatively high were:

1. Item number 11: “I always know where I am with regard to where I live”.

Experimental group 4.02 (SD=1.084), control group 4.29 (SD=1.141) and total average 4.16 (SD=1.115)

2. Item number 8: “I have no problems parking a car (in case you are not driving, please leave empty)”

Experimental group 3.85 (SD=1.085), control group 4.29 (SD=1.019) and total average 4.07 (SD=1.068)

3. Item number 16: “I have good balance and eye-hand coordination and enjoy sports which use a ball”.

Experimental group 3.76 (SD=1.179), control group 4.09 (SD=1.145) and total average 3.93 (SD=1.166)

4. Item number 9: “Use of numbers and symbols is easy for me”.

Experimental group 3.63 (SD=1.968), control group 4.16 (SD=1.186) and total average 3.91 (SD=1.113)

5. Item number 15: “Knowing directions is easy for me”.

Experimental group 3.78 (SD=1.107), control group 3.96 (SD=1.086) and total average 3.87 (SD=1.093)

Subjects have also been asked to include the average grade they had in mathematics in high school. From the results an almost even distribution between grades two to four is observable, with an additional discrepancy in the percentage of subjects with average grade five between the experimental and control group (17.1% of subjects in the experimental and 31.0% of subjects in the control group). On the other hand 29.3% of subjects in the experimental group stated their average mathematics high school grade was two whereas only 11.9% percent of subjects in the control group were awarded the same grade. Highest percentage of subjects in the experimental group were awarded grades two and three (29.3%), whereas highest percentage of subjects in the control group were awarded grade five (31.0%). Lowest percentage of subjects in the experimental group were awarded grade five (17.1%) whereas lowest percentage of subjects in the control group were awarded grade two (11.9%). Total highest percentage of subjects have been awarded grade three (28.9%) and lowest total percentage of subjects have been awarded grade two (20.5%).

Variable	Group	Total			2,00	3,00	4,00	5,00
		N	M	SD	%	%	%	%
Math grade	Experimental	41	3,29	1,078	29,3%	29,3%	24,4%	17,1%
	Control	42	3,79	1,025	11,9%	28,6%	28,6%	31,0%
	Total	83	3,54	1,074	20,5%	28,9%	26,5%	24,1%

Table 9. Average grade in mathematics in high school

8.6 Analyses

Since spatial abilities are one aspect of the general intelligence, t-test for independent samples has been run with regard to general intelligence.

Group	N	M	SD	t	df	p
Experimental	49	43.204	14.741	-3.842	78.3*	0.000
Control	44	52.682	8.526			

*Correction due to unequal variance

Table 10. T-test for independent samples with regard to general intelligence

It can be concluded that the t-test value is greater than the boundary value with p=0.001 which implies the existence of statistically significant difference between the scores of the experimental and control group on the spatial abilities test.

Due to the obvious higher results with regard to the control group ANCOVA analysis has been conducted with intelligence as covariance, in order to make experimental and control group equal with regard to the first point which would enable comparison.

Group	N	M	M_e	F	df	p	η_p^2
Experimental	39	55.89	59.159	54.572	1	0.000	0.406
Control	44	53.36	50.473				

M_e – Estimated mean after controlling initial result in intelligence
η_p^2 – partial eta squared

Table 11. ANCOVA results

Results of the analysis imply that after initial difference with regard to intelligence there was a significant difference between the results of the experimental and control group. However no statistically significant differences have been found with regard to gender (F=3.904; df=1; p=.052) and between the two groups and gender (F=1.688; df=1; p=.198).

T-test for dependent samples has also been conducted in order to check differences within groups and examine whether there are differences between initial and final testing.

Group	Test	N	M	SD	t	df	p_t	r	p_r
Experimental	Initial Smith & Whetton	39	46.28	11.116	-9.126	38	.000	.833	.000
	Final Smith & Whetton	39	55.90	11.571					
Control	Initial Smith & Whetton	44	52.68	8.526	-1.431	43	.160	.952	.000
	Final Smith & Whetton	44	53.36	9.886					

t,df,p_t – paired samples t-test
r,p_r – correlation parameters

Table 12. T-test for dependent samples

Results show a statistically significant jump in scores with regard to the initial and final testing with the experimental group t=-9,126, df=38, p<0,05), whereas the control group did not exhibit statistically significant differences between initial and final scores.

The correlation between individual items on the questionnaire and the initial spatial abilities test by Smith and Whetton (1988) is outlined in table.

Item	Correlation with total initial Smith & Whetton spatial test score	Correlation with experimental group initial Smith & Whetton spatial test score	Correlation with control group initial Smith & Whetton spatial test score
1. When I was a child I enjoyed playing games which included putting different pieces together (Lego blocks, jigsaw puzzles, 3D models etc).	.265*	.279	.410*
2. When I was a child I enjoyed solving Sudoku, labyrinths and 'connect the dots' problems.	.196	.080	.297
3. When I was a child I loved to play chess.	.008	-.187	.176
4. I was good at science subjects at school.	.125	.094	.058
5. I was in scouts when I was a child.	.177	.124	.263
6. When I was a child I spent a lot of time outdoors.	-.045	.096	-.181
7. When I am walking around unfamiliar city I can easily retrace my steps back to the starting point.	.172	.204	.078
8. I have no problems parking a car (in case you are not driving, please leave empty).	.050	.114	-.160
9. Use of numbers and symbols is easy for me.	.051	.077	-.097
10. Music is very important to me in everyday life.	.008	.055	-.100
11. I always know where I am with regard to where I live.	.077	.074	.023
12. I often develop equations in order to describe correspondences or explain what I see.	.119	.018	.095
13. I do not get lost easily and the use of maps and landmarks is easy for me.	.152	.153	.151
14. I have a very good sense of pitch, tempo and rhythm.	-.294*	-.410**	-.143
15. Knowing directions is easy for me.	.070	.043	.065
16. I have good balance and eye-hand coordination and enjoy sports which use a ball.	-.042	.068	-.266
17. I have the ability to represent what I see by drawing or painting.	.000	-.166	.072
18. My excellent coordination and balance enable me to be very successful in activities which demand high speed.	-.066	-.057	-.137
19. I like to think about numerical issues and examine statistics.	.210	.110	.210
20. My ability to draw is recognised and complimented by others.	.015	-.114	.021
21. I can remember the tune of a song when asked.	-.256*	-.220	-.375*
22. It is easy for me to visualise things in 3D.	.290*	.389*	.133
Total questionnaire	.116	.410*	.279

*p<.05, **p<0.01

Table 13. Correlation between individual questionnaire items and initial spatial test

With regard to the *total scores* on the initial Smith and Whetton (1988) spatial test, the following correlations have been found:

1. Item number 1 ("When I was a child I enjoyed playing games which included putting different pieces together (Lego blocks, jigsaw puzzles, 3D models etc)" with positive correlation r=0.265 (p<.05)

2. Item number 14 ("I have a very good sense of pitch, tempo and rhythm") with negative correlation r=-0.294 (p<.05)

3. Item number 21 ("I can remember the tune of a song when asked") with negative correlation r=-0.256 (p<.05)

Overall correlation between the questionnaire and the total score on the initial Smith and Whetton (1988) spatial test was 0.116, what is statistically insignificant.

With regard to the *experimental group* specifically, the following correlations between individual items in the questionnaire and experimental group score have been found:

1. Item number 14 ("I have a very good sense of pitch, tempo and rhythm.") with negative correlation r=-0.410 (p<.01)

2. Item number 22: ("It is easy for me to visualise things in 3D.") with positive correlation r=0.389 (p<.05)

Overall statistically significant correlation between the questionnaire and the experimental group score on the initial Smith and Whetton (1988) spatial test was r=0.410 (p<.05).

With regard to the control group specifically, the following correlations between individual items in the questionnaire and experimental group score have been found:

1. Item number 1 (“When I was a child I enjoyed playing games which included putting different pieces together (Lego blocks, jigsaw puzzles, 3D models etc)”) with positive correlation r=0.410 (p<.05).

2. Item number 21 (“I can remember the tune of a song when asked”) with negative correlation r=-0.375 (p<.05).

Overall correlation between the questionnaire and the control group score on the initial Smith and Whetton (1988) spatial test was 0.279, what is statistically insignificant.

In the following table correlations between tests and average mathematics grade in high school have been outlined.

		Initial Smith & Whetton	Final Smith & Whetton	Final Newton & Bristoll
Initial Smith & Whetton	Total	1		
	Kont	1		
	Exp	1		
Final Smith & Whetton	Total	.795**	1	
	Kont	.952**	1	.158
	Exp	.833**	1	.576**
Final Newton & Bristoll	Total	.394**	.395**	1
	Kont	.316	.158	1
	Exp	.474**	.576**	1
Avg math grade	Total	.168	.093	.272*
	Kont	.257	.215	.167
	Exp	-.013	.062	.392*

*p<.05, **p<0.01

Table 14. Correlation between tests and average mathematics grade in high school

With regard to the *total scores*, the following correlations have been found between:

1. Initial Smith and Whetton (1988) spatial test and final Smith and Whetton (1988) spatial test with r=0.795 (p<0.01)

2. Initial Smith and Whetton (1988) spatial test and final Newton and Bristoll (2009) spatial test with r=0.394 (p<0.01)

3. Final Smith and Whetton (1988) spatial test and final Newton and Bristoll (2009) spatial test with r=0.395 (p<0.01)

4. Final Newton and Bristoll (2009) spatial test and average mathematics grade in high school with r=0.272 (p<0.05)

With regard to the *experimental group* specifically, the following correlations have been found between:

1. Initial Smith and Whetton (1988) spatial test and final Smith and Whetton (1988) spatial test with r=0.833 (p<0.01)

2. Initial Smith and Whetton (1988) spatial test and final Newton and Bristoll (2009) spatial test with r=0.474 (p<0.01)

3. Final Smith and Whetton (1988) spatial test and final Newton and Bristoll (2009) spatial test with r=0.576 (p<0.01)

4. Final Newton and Bristoll (2009) spatial test and average mathematics grade in high school with r=0.392 (p<0.05)

With regard to the *control group* specifically, the following correlation has been found between:

1. Initial Smith and Whetton (1988) spatial test and final Smith and Whetton (1988) spatial test with r=0.952 (p<0.01)

8.7 Discussion and conclusion

The research which has been described within the scope of this thesis has run across many difficulties of both organisational as well as technical nature, which need to be taken into consideration when considering the results of the research and comparing them with the results of previous, similar researches. Researches which entail a specific experimental programme running across a longer period of time are complicated to organise in practice because the subjects involved need to see specific gains from it and also it is necessary to manage to fit the programme into the frames of the subjects' schedule. Technical details also need to be taken into consideration such as materials available in the classroom where the programme would be running, which in this case were computers on which GeoGebra 5 was installed, as well as the materials provided by the researcher conducting the programme such as .ppt presentations, wooden models with which the subjects were going to work etc. Also, one of the main concerns of researchers conducting this type of research running across several weeks is how many subjects would attend all of the lessons and how many would skip some of them, because in cases in which final testing is measuring the effect of the whole experimental programme the subjects who skipped some of them cannot be taken into consideration and they need to be excluded from the sample, which however can only be reviewed at the end of the research.

As it is often the case in STEM fields, male students are predominant and researchers need to take this into consideration when analysing and interpreting the data. In our case the number of male subjects roughly exceeded the number of female subjects in both groups (35 male and 17 female in experimental and 33 male and 19 female in control group). The fact that researches similar to our experimental programme have continually proved gender inequality with regard to the results on the spatial tests in favour of men (Eals & Silverman, 1994; Harris, 1978; Linn & Petersen, 1986; Hoyenga & Hoyenga, 1998) needs to be measured against our own findings which indicate no statistically significant differences have been found either with regard to gender ($F=3.904$; $df=1$; $p=.052$) or between the experimental and control groups and gender ($F=1.688$; $df=1$; $p=.198$). It can therefore be stated that auxiliary hypothesis 1, i.e. 'there are no statistically significant differences between the genders neither with regard to the experimental programme (initial and final testing) nor with regard to the control group' has been corroborated. Although these results are inconsistent with the results of previous researches with equivalent aims (Sorby et al, 2014; Sorby & Baartmans, 2000) it needs to be taken into consideration the small sample size used for our research, in which case clustering is probable. Continuing along this line are also differences in scores on the initial Smith and Whetton (1988) spatial abilities test between the experimental and control group (average score of 43.20 for the experimental and 52.68 for the control group), which discrepancy was corrected with the application of ANCOVA analysis in order to enable the comparison and analysis. Average score was 55.89 for the experimental and 53.36 for the control group ($F=54.572$, $df=1$, $p=0.000$, $\eta_p^2=0.406$). It can be stated therefore with regard to hypothesis 1, i.e. 'differences between the control and experimental group at initial testing of spatial abilities are not statistically significant with regard to any subsample' that the hypothesis has been corroborated.

Similarly, with regard to the background questionnaire and the initial Smith and Whetton (1988) test no overall statistically significant correlations have been found ($r=.116$), although correlations have been found with regard to the individual items, i.e. Item number 1 ("When I was a child I enjoyed playing games which included putting different pieces together (Lego blocks, jigsaw puzzles, 3D models etc)" with positive correlation $r=0.265$ ($p<.05$), item number 14 ("I have a very good sense of pitch, tempo and rhythm") with negative correlation $r=-0.294$ ($p<.05$) and item number 21 ("I can remember the tune of a song when asked") with negative correlation $r=-0.256$ ($p<.05$). Previous researches have in cases included similar questionnaires with the aim of isolating possible predictors of highly developed spatial abilities (Sorby & Baartmans, 2000) and correlations have been found between 1) play as children with construction toys, 2) gender, 3) math scores, 4) previous experience in design-related courses like drafting, mechanical drawing etc. (Sorby & Baartmans, 2000), what in our case has not been confirmed. The correlation between average grade in mathematics in high school and the scores on the initial Smith and Whetton spatial abilities test has not been significant ($r=.168$).

Results show that with the experimental group a statistically significant jump in performance on the post-test has occurred ($t=-9.126$, $df=38$, $p<0.05$), whereas with regard to the control group no statistically significant changes in performance have been noted ($t=-1.431$, $df=43$, $p>0.05$). Therefore it can be stated that hypothesis 2, 'differences between the control and experimental group at final testing of spatial abilities are statistically significant' has been corroborated.

Statistically significant correlation between the scores of the experimental group on the initial and final Smith and Whetton (1988) spatial abilities test imply a significant jump in performance, which in such measure we can only attribute to the effect of the experimental programme ($r=0.833$; $p<0.01$). It can therefore be stated that hypothesis 4, i.e. 'differences between initial and final testing of spatial abilities within the experimental group are statistically significant' has been corroborated. Statistically significant correlation between the scores of the control group on the initial and final Smith and Whetton (1988) test has also been found ($r=0.952$; $p<0.01$). Although the difference in their average scores is not as large as that of the experimental group, this can perhaps be explained through the training effect. However, it has been empirically proved that it is not possible to create such large differences in scores by employing the test training method (Zarevski, 2000). It can therefore be stated that hypothesis 3 'differences between initial and final testing of spatial abilities within the control group are not statistically significant' has been only partially corroborated.

Statistically significant correlation has also been found between the scores of the experimental group on the initial Smith and Whetton (1988) and final Newton and Bristoll (2009) spatial test ($r=0.474$; $p<0.01$), as well as between the final Smith and Whetton (1988) and final Newton and Bristoll (2009) test ($r=0.576$; $p<0.01$), which can also be explained through the effect of the training programme. It is interesting to note that statistically significant correlations between the average mathematics grade in high school and any scores of any of the two groups has been found between the total score on the final Newton and Bristoll (2009) spatial test and average mathematics grade ($r=0.272$; $p<0.05$) and between the score of the experimental group specifically ($r=0.392$; $p<0.05$), which however can be disregarded since taking into consideration scores on the final test is pointless at best. It can therefore be stated that auxiliary hypothesis 2, i.e. 'knowledge gained during high school education does not have a statistically significant impact on the differences between the students' success within the experimental group' has not been corroborated.

As a research with a very small sample and in conditions in which technical and organisational difficulties needed to be abridged, it can be pointed out that the results suggest there is a fertile ground for a similar research on a larger sample and running across a longer period of time. Results have showed that the developed experimental programme, even in the conditions in which it has been run, harbours potential for expansion and further application. Even if we take into consideration that the obtained results imply that training effect has taken place, the findings suggest there is a lot of room for spatial abilities training using

GeoGebra 5 as a tool within a short span of time and without employing complex techniques or tasks which the subjects might find discouraging. The exercises employed in the developed experimental programme have been well received, the mathematical problems presented were on the appropriate level of difficulty and GeoGebra itself offered tools which enabled them to understand the difficulties they had with previous tasks, as well as learn from the experience. The rotate tool has proved to be highly useful, as well as different methods of sketching which the software offers, so that each individual can choose which method to employ in order to solve the problem according to his or her preferences and inclinations. The level of difficulty of the problems presented can easily be adjusted by adding more complex problems in case the subjects show more aptitude at solving them but also because the problems have been created and presented according to the scaffolding rule in case the subjects start showing increased difficulties in solving the more complex ones they can easily be removed from the programme, which in general should not affect the effect of the experimental programme.

The results obtained in this research need to be viewed through the prism of the conditions it has been run in, take into consideration the small size of the sample, the proportions between the number of male and female subjects and also the time period throughout which it has been running. With all this taken into consideration, along with the obtained results, it can concluded that with the expansion of the experimental programme, larger sample and across a longer period of time only better results can be expected.

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10 Appendix 1 – Questionnaire

UPITNIK

Ime i prezime

Spol

Dob

Fakultet/smjer

Prosječna ocjena iz matematike u srednjoj školi

Poštovani, pred Vama je upitnik s nizom tvrdnji. Molimo Vas da za svaku od njih procijenite u kojoj mjeri se odnosi na Vas, pri čemu brojevi 1-5 znače slijedeće:

1-uopće se ne odnosi na mene
2-uglavnom se odnosi na mene
3-niti se odnosi, niti se ne odnosi na mene
4-uglavnom se odnosi na mene
5-u potpunosti se odnosi na mene

1.	U djetinjstvu sam volio/la se igrati igara koje uključuju slaganje manjih elemenata u cjelinu (npr. Lego-kocke, puzzle, 3D makete i sl.)	1	2	3	4	5
2.	U djetinjstvu sam volio/la riješavati Sudoku, labirinte i „spoji točkice".	1	2	3	4	5
3.	U djetinjstvu sam volio-la igrati šah.	1	2	3	4	5
4.	U školi sam bio-la dobar/dobra u znanstvenim predmetima.	1	2	3	4	5
5.	U djetinjstvu sam bio-la u izviđačima.	1	2	3	4	5
6.	U djetinjstvu sam puno vremena provodio/la na otvorenom, u prirodi.	1	2	3	4	5
7.	U situaciji kada po prvi put šetam nepoznatim gradom lako se uspijem vratiti na početnu točku.	1	2	3	4	5
8.	Lako parkiram automobil (ukoliko niste vozač, molimo ostaviti prazno).	1	2	3	4	5
9.	Upotreba brojeva i simbola je laka za mene.	1	2	3	4	5
10.	Glazba mi je važna u svakodnevnom životu.	1	2	3	4	5
11.	Uvijek znam gdje se nalazim u odnosu na mjesto gdje živim.	1	2	3	4	5
12.	Često smišljam jednadžbe kako bih opisao/la povezanosti ili objasnio/la ono što vidim.	1	2	3	4	5
13.	Ne mogu se lako izgubiti i lako se orijentiram prema kartama i orijentirima u prostoru.	1	2	3	4	5
14.	Imam dobar osjećaj za visinu tona, tempo i ritam.	1	2	3	4	5

15. Lako se orijentiram u prostoru.	1	2	3	4	5
16. Imam dobru ravnotežu i koordinaciju između oka i ruke te uživam igrati sportove sa loptom.	1	2	3	4	5
17. Imam sposobnost prikazati ono što vidim crtanjem ili slikanjem.	1	2	3	4	5
18. Moja izvanredna koordinacija i ravnoteža mi omogućuju da budem što uspješniji/la u aktivnostima koje zahtijevaju veliku brzinu.	1	2	3	4	5
19. Volim razmišljati o numeričkim problemima i proučavati statistiku.	1	2	3	4	5
20. Moja sposobnost crtanja je prepoznata i cijenjena od strane drugih.	1	2	3	4	5
21. Mogu reproducirati melodiju pjesme na zahtjev.	1	2	3	4	5
22. Lako mi je predočiti si stvari trodimenzionalno.	1	2	3	4	5

11 Author's CV

The author has been born on the 25th of November, 1981 in Zagreb, Croatia. She graduated from the Faculty for Natural Sciences, Department for Mathematics, University of Zagreb, in 2010 with the thesis 'Calculating coefficients in the two-phase, compressible flow model through porous media in formulation with global pressure' under the mentorship of prof.dr.sc. I. Jurak. Since 2010 until 2011 the author worked as TA at the College for Finance and Law Effectus, Zagreb. Since 2011 until 2017 the author worked as TA at the Faculty for Teacher Education, University of Zagreb, Zagreb.

Published papers:

1. Tomić, M. K., Šimović, V., & Miloloža, I. (2011, January). Financial Management Control as Methodology of Development of Information Systems for Business Management. In *InterSymp-2011. The 23rd International Conference on Systems Research, Informatics and Cybernetics and the 31st Annual Meeting of the IIAS. Pre-Conference Proceedings of the Special Focus Symposium on 11th ICESAKS.*

2. Kopačević, D., Rogulja, N., & Tomić, M. K. (2011). Flow experience among future teachers during studies. *Croatian journal of education*, *13*(4), 175-195.

3. Nataša, R., Tomić, M. K., & Kopačević, D. (2011, January). Flow in Computer Human Environment. In *The 23rd International Conference on Systems Research, Informatics and Cybernetics and the 31st Annual Meeting of the IIAS.*.

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12 Povzetek

Razvoj in validacija kurikuluma za prostorsko usposabljanje z uporabo programske opreme za dinamično geometrijo na nivoju univerzitetnega izobraževanja

Prostorske sposobnosti, ki jih Linn in Petersen opisujeta kot "spretnost za zastopanje, preoblikovanje, ustvarjanje in priklic simboličnih nejezikovnih informacij" (Linn & Petersen, 1985), se kot take ne poučujejo v šolah. Prepušča se jih naravnemu razvoju in s pomočjo nekaterih dejavnosti v otroštvu, ki so se pokazale, da so prediktorji zelo razvitih prostorskih sposobnosti (Sorby & Baartmans, 2000). Kljub temu je bil pomen visoko razvitih sposobnosti študentov STEM (znanosti, tehnologije, inženiringa in matematike) znanstveno dokazan (npr. Gohm, Humphreys, & Yao, 1998; Humphreys, Lubinski, & Yao, 1993; Lohman, 1988, 1994a, 1994b; Smith, 1964), kar neposredno nakazuje, da lahko prostorske sposobnosti pomembno vplivajo na uspeh ne le študija predmetov STEM, temveč tudi na uspeh v STEM karieri.

Skozi leta so raziskovalci prepoznali vrednost in povezavo med uspešnostjo na področjih STEM z visoko razvitimi prostorskimi sposobnostmi. Zato so z namenom pomagati študentom STEM razviti njihove prostorske sposobnosti, poskušali razviti vaje kot tudi celotne eksperimentalne programe za izboljšanje prostorskih sposobnosti (Sorby & Baartmans, 2000; Martín-Dorta, Saorín & Contero, 2008; Güven & Kosa, 2008). Njihova uspešnost je bila potrjena s pozitivno korelacijo med prostorskimi predtesti in potesti. V disertaciji je v nadaljevanju predstavljen novo razvit eksperimentalni program za izboljšanje prostorskih sposobnosti.

Za predtest je bil uporabljen prostorski test Pauline Smith in Chrisa Whettona (Smith & Whetton, 1988). Po končanem eksperimentalnem programu je bil test ponovno uporabljen, skupaj z vprašalnikom, ki je vseboval 22 trditev, ki obravnavajo možne prediktorje visoko razvite prostorske spretnosti z uporabo Likertove lestvice. Vprašalnik v glavnem vsebuje elemente iz vprašalnika, ki ga je napisal dr. Terry Armstrong, medtem ko so bile dodane nekatere trditve, ki so jih drugi raziskovalci preizkusili kot prediktorje za visoko razvite prostorske sposobnosti, skupaj z nekaterimi trditvami, ki sta jih Sorby in Baartmans (2000) označila kot visoko razvite prediktorje prostorskih sposobnosti. Poleg omenjenih vprašanj so bila vključena v vprašalnik vprašanja o starosti, spolu in povprečni oceni matematike v srednji šoli. Poleg tega sta bila opravljena še preizkus prostorskih sposobnosti Newton in Bristoll (2009).

Glavna hipoteza naše raziskave je bila: *„prostorske sposobnosti je mogoče izboljšati s posebej razvitim programom z uporabo računalniškega programa Geogebra v obdobju najmanj enega meseca. Glede na prejšnje raziskave bodo izboljšane prostorske sposobnosti študentom predstavljale temelj za uspeh pri njihovem študiju in tudi v prihodnosti*. " Pomožne hipoteze so bile: 1) Med spoloma ni statistično pomembnih razlik glede na eksperimentalni program (začetno in končno testiranje), niti ni razlik glede na kontrolno skupino; 2) Znanje, pridobljeno med srednješolskim izobraževanjem, nima statistično pomembnega vpliv na razlike med uspešnostjo študentov v eksperimentalni skupini (stalno spremljanje napredovanja med eksperimentalnim programom ter začetno in končno testiranje).

V naši raziskavi je bil uporabljen računalniški program GeoGebra kot glavno orodje za razvijanje prostorskih veščin. Eksperimentalno skupino je sestavljalo 35 moških (67,3%) in 17 žensk (32,7%), medtem ko je bilo v kontrolni skupini 33 moških (63,5%) in 19 žensk (36,5%). Vsi udeleženci so študirali na Fakulteti za naravoslovje in izobraževanje v Mostarju, Bosna in Hercegovina, s tem da so obiskovali različne študijske programe. Eksperimentalna skupina se je udeležila eksperimentalnega programa, medtem ko se je kontrolna skupina udeleževala običajnega študijskega procesa. Predtest in potest sta bila izvedena skoraj istočasno v eksperimentalni in kontrolni skupini. Za statistično analizo smo za odvisne spremenljivke določili uspeh študentov med eksperimentalnim programom in rezultate študentov glede prostorske sposobnosti v pred in potestu, neodvisna spremenljivka je bila spol udeležencev.

Za eksperimentalno skupino, ki je bila razdeljena v dve enaki skupini, je bil zasnovan eksperimentalni program s predvidenim trajanjem štirih tednov. Vnaprej je bilo odločeno, da poskusne skupine ne smejo biti večje od 20 do 25 študentov, da bi tako učitelj lahko nadzoroval pravilno izvedbo vseh korakov eksperimentalnega programa. Dodatno smo se odločili, da bo v eksperimentalnem programu sodelovalo največ 60 študentov, vendar ne manj kot 35 študentov. Razlog za tako majhen vzorec je, da programa raziskovalka ne bi samo predavala in pregledala študentske izdelke po zaključku vsake vaje, temveč morajo predavanja slediti določenim korakom: 1) študent mora vizualizirati dani problem, 2) študent ga mora skicirati s pomočjo metode pisala in papirja, 3) študent modelira problem v GeoGebri in 4) študentje izvedejo sami reševanje dodatne problemov in individualno analizo podobnosti in razlik med računalniškimi in papirnatimi skicami za napredovanje njihovih prostorskih sposobnosti. Cilj teh štirih korakov je bil, da študentje najprej ustvarijo miselno podobo danega problema, skicirajo svojo miselno podobo s pomočjo pisala in papirja, šele nato pa s pomočjo programske opreme modelirajo dani probem, tako da lahko na koncu primerjajo tisto, kar so pred tem vizualizirali, s sliko, ki so jo dobili z računalniškim programom GeoGebra.

Kontrolna skupina je bila skupina študentov v prvem letniku in je študirala na isti univerzi kot eksperimentalna skupina, vendar se niso udeležili eksperimentalnega programa. Kontrolna skupina je nadaljevala s svojimi običajnimi dejavnostmi in ni bila podvržena kakršnemu koli vplivu raziskave. Na kratko, oba študenta, razvrščena v eksperimentalni in v kontrolni skupini, ki sta se učila na isti univerzi, sta bila iste starosti in sta prišla približno z enakim znanjem iz srednjih šol. Edina razlika med njima je, da jih je polovica sodelovala v eksperimentalnem programu, druga polovica pa ne.

V prvem tednu eksperimentalnega programa so se študentje uvajali v delo s računalniškim programom GeoGebra 5, tj. učenje uporabe 2D in 3D pogleda, funkcij ukaznega polja in sintakse. Po tem, ko so se študenti seznanili z osnovnimi ukazi v GeoGebri, je sledila vizualizacija in nato skiciranje geometrijskih predmetov, kar je služilo kot priložnost za uporabo osnovnega znanja, ki so ga pridobili v tem uvodnem predavanju.

V drugem tednu eksperimentalnega programa so študenti delali z lesenimi predmeti. Vsak študent je dobil en lesen predmet, s katerim je delal. Najprej so dobili navodilo, da opravijo meritve predmeta in ga skicirajo na papir, potem pa morajo meritve prevesti v kartezične koordinate in objekt modelirati v GeoGebri. Po modeliranju lesenih predmetov v GeoGebri so bile predstavljene ortogonalne skice lesenih predmetov, s katerimi študentje še niso delali in so vsebovale sprednji, stranski in zgornji pogled. Naloga študentov je bila te predmete modelirati v GeoGebri. Podana je bila kratka razlaga o tem, kaj vsebujejo ortogonalne skice in opredelitve vsakega pogleda. Po razlagi so študentje nadaljevali z nalogo. Študentje so pri tej nalogi bili bistveno hitrejši in učinkovitejši kot pri prejšnji nalogi, razlogi za to bi lahko bili: 1) pridobivali so izkušnje med prevajanjem meritev predmetov v kartezične koordinate, ki naj bi jih modelirali v GeoGebri, 2) modeliranje v GeoGebri in vrtenje njihovega modela predmeta je pomagalo pri postopku vizualizacije, ki je bil potreben tudi za to nalogo. Ena od olajševalnih okoliščin je lahko tudi, da so bili predmeti konceptualno podobni, zato bi lahko vaja za prvo nalogo bistveno olajšala študentom pri reševanju druge naloge.

V tretjem tednu eksperimentalnega programa so bile vaje iz vrtenja in zrcaljenja v 2D. Geogebra apleti so bili pripravljeni in naloženi na GeoGebraTube, do katerih so študenti lahko dostopali in z njimi delali. Vse skupaj so bile tri vaje za zrcaljenje in tri za vrtenje. V vajah zrcaljenja je bilo od študentov zahtevano, da zrcalijo vse točke glede na črto in jih povežejo v poligon. Pravilno rešitev so lahko preverili v Geogebri z orodjem 'zrcali preko točke'. Pri vajah iz vrtenja je bilo potrebno vrteti dani poligon za določen kot okrog dane točko, ki je bila edina označena točka. Pravilno rešitev je bilo mogoče preveriti v GeoGebri z orodjem 'vrti okoli točke'. Nekateri študentje so vse naloge opravili v tridesetih minutah, nekateri pa so jih težko opravili v eni uri in trideset minutah. Nekateri študenti so imeli tehnične težave, zaradi česar je reševanje nalog potekalo počasneje.

Za zadnji teden eksperimentalnega programa so bile zasnovane matematične naloge s poudarkom na vizualizaciji, spet po štiristopenjski metodi. Vaje smo razdelili na dva dela, prvi nivo (osnovni) in drugi nivo (napredni). Za vsako matematično nalogo so morali študenti najprej vizualizirati problem (prvi korak) in ga narisati na papir (drugi korak), nato so ga modelirali v GeoGebri (tretji korak) in nazadnje primerjati svojo začetno skico na papirju s tisto, ki je bila modelirana v GeoGebri (četrti korak). Naloga druge stopnje je vključevala dodatno nalogo, ki je bila namenjena študentom, ki so že zaključili naloge osnovne ravni.

Na predtestu Smith in Whetton (1988) je eksperimentalna skupina dosegla povprečje 43,20 (najslabši rezultat je bil 12 in najboljši rezultat 69) in standardni odklon 14,74, medtem ko je kontrolna skupina dosegla povprečno oceno 52,68 (z najslabšim rezultatom 36, najboljši rezultat pa 71) in standardni odklon 8,53. Treba je opozoriti na opazno večje standardno odstopanje za rezultate eksperimentalne skupine na predtestu. Na potestu Smith in Whetton (1988) je eksperimentalna skupina dosegla povprečno vrednost 55,93 (najslabši rezultat 22 in najboljši rezultat 76) in standardni odklon 11,42, kontrolna skupina pa povprečno oceno 53,36 (najslabši rezultat je bil 37 in najboljši rezultat 73), standardni odklon pa 9,89. Opazimo manj izrazite razlike v standardnih odklonih v medsebojni primerjavi. V preskusu prostorskih sposobnosti (Newton in Bristoll, 2009), ki je bil opravljen kot potest, je eksperimentalna skupina dosegla povprečno vrednost 30,54 (z 18 od največ 45 kot najslabši rezultat in 40 kot najboljši rezultat) s standardnim odklonom 5,07. Na istem testu je kontrolna skupina dosegla povprečni rezultat 31 (13 od največ 45 kot najslabši rezultat in 43 kot najboljši rezultat) s standardnim odklonom 5,89.

Raziskave, podobne našemu eksperimentalnemu programu, nenehno dokazujejo neenakost med spoloma glede na rezultate prostorskih testov v korist moških (Eals & Silverman, 1994; Harris, 1978; Linn & Petersen, 1986; Hoyenga & Hoyenga, 1998). Naše ugotovitve kažejo, da ni bilo statistično pomembnih razlik bodisi glede na spol ($F = 3.904$; $df = 1$; $p = .052$) bodisi med eksperimentalno in kontrolno skupino ter spolom ($F = 1.688$; $df = 1$; $p = .198$). Zato lahko trdimo, da je bila pomožna hipoteza 1, tj. *„Ni statistično pomembnih razlik med spoloma niti glede eksperimentalnega programa (začetno in končno testiranje) niti glede kontrolne sku*pine“, potrjena. Čeprav ti rezultati niso v skladu z rezultati prejšnjih raziskav z enakimi cilji

(Sorby in sod., 2014; Sorby & Baartmans, 2000), je treba upoštevati majhno velikost vzorca, uporabljeno za naše raziskave, saj je v tem primeru verjetno grozdenje.

V to smer se nadaljujejo tudi razlike v ocenah na predtestu prostorskih sposobnosti (Smith in Whetton, 1988) med eksperimentalno in kontrolno skupino (povprečna ocena 43,20 za eksperimentalno in 52,68 za kontrolno skupino), kjer je bilo odstopanje odpravljeno z analizo ANCOVA, da bi omogočili primerjavo in analizo. Povprečna ocena je bila 55,89 za eksperimentalno in 53,36 za kontrolno skupino ($F = 54{,}572$, $df = 1$, $p = 0{,}000$, $\eta p2 = 0{,}406$). V zvezi s hipotezo 1 je torej mogoče trditi, da „razlike med kontrolno in eksperimentalno skupino pri začetnem preskušanju prostorskih sposobnosti niso statistično pomembne za kateri koli podvzorec“, zato je bila hipoteza potrjena.

Podobno, v zvezi z vprašalnikom o predhodnih izkušnjah in predtestom Smith in Whetton (1988) niso bile ugotovljene splošne statistično pomembne korelacije ($r = .116$), čeprav so bile ugotovljene korelacije glede na posamezne postavke, trditev 1: "V otroštvu sem užival v igranju iger, ki so vključevale sestavljanje različnih kosov (Lego bloki, sestavljanke, 3D modeli itd.)" s pozitivno korelacijo $r = 0{,}265$ ($p <.05$); trditev št. 14: "zelo imam dober občutek za ton, tempo in ritem ”) z negativno korelacijo $r = -0{,}294$ ($p <.05$) in trditev 21: " Če me kdo vpraša, se spomnim melodije skladbe " z negativno korelacijo $r = -0{,}256$ ($p <.05$). Prejšnje raziskave so vključevale podobne vprašalnike z namenom izoliranja možnih prediktorjev visoko razvitih prostorskih sposobnosti (Sorby & Baartmans, 2000), kjer so bile ugotovljene korelacije med 1) igranjem otrok z konstrukcijskimi igračami, 2) spolom , 3) matematičnimi ocenami, 4) prejšnje izkušnje s tečajev, povezanih z oblikovanjem, kot so skiciranje, strojno risanje, itd., kar v našem primeru ni bilo potrjeno. Povezava med povprečno oceno matematike v srednji šoli in rezultati na začetnem testu prostorskih sposobnosti Smith in Whetton ni bila signifikantna ($r = .168$).

Rezultati kažejo, da je pri eksperimentalni skupini prišlo do statistično pomembnega skoka zmogljivosti na potestu ($t = -9.126$, $df = 38$, $p <0{,}05$), medtem ko v zvezi s kontrolno skupino statistično pomembnih sprememb v uspešnosti ni bilo opaziti ($t = -1.431$, $df = 43$, $p> 0{,}05$). Zato lahko trdimo, da je bila hipoteza 2, „razlike med kontrolno in eksperimentalno skupino pri končnem preskušanju prostorskih sposobnosti so statistično pomembne“, potrjena.

Statistično značilna korelacija med točkami eksperimentalne skupine na začetnem in končnem preskusu prostorskih sposobnosti Smith in Whetton (1988) pomeni znaten skok v uspešnosti, ki ga v takšnem merilu lahko pripišemo le učinku eksperimentalnega programa ($r = 0{,}833$; $p <0{,}01$). Zato lahko trdimo, da je bila potrjena hipoteza 4, to je „razlike med začetnim in končnim preskušanjem prostorskih sposobnosti v eksperimentalni skupini so statistično pomembne“. Ugotovljena je bila tudi statistično pomembna povezava med rezultati kontrolne skupine na začetnem in končnem testu Smith in Whetton (1988) ($r = 0{,}952$; $p <0{,}01$). Čeprav razlika v njihovih povprečnih ocenah ni tako velika kot pri poskusni skupini, je to mogoče razložiti z učinkom treninga. Vendar pa je empirično dokazano, da ni mogoče ustvariti tako velikih razlik v rezultatih z uporabo metode testnega treninga (Zarevski, 2000). Zato lahko trdimo, da je hipoteza 3, „razlike med začetnim in končnim preskušanjem prostorskih sposobnosti v kontrolni skupini statistično nepomembna“, le delno potrjena.

Ugotovljena je bila tudi statistično pomembna korelacija med rezultati eksperimentalne skupine na začetnem testu Smith in Whetton (1988) ter končnem prostorskem testu Newtona in Bristola (2009) ($r = 0{,}474$; $p <0{,}01$) ter med končnim testom Smith in Whetton (1988) ter končnim testom Newton in Bristol (2009) (r = 0,576; p <0,01), kar je mogoče razložiti tudi z učinkom eksperimentalnega programa. Zanimivo je, da smo med skupno oceno na končnem prostorskem preizkusu Newtona in Bristoll (2009) in povprečno oceno matematike našli statistično pomembne korelacije med povprečno oceno matematike v srednji šoli in vsemi rezultati katere koli od obeh skupin. (r = 0,272; p <0,05) in posebej med oceno eksperimentalne skupine (r = 0,392; p <0,05), ki pa je ni mogoče upoštevati, saj je upoštevanje rezultatov na končnem testu v najboljšem primeru nesmiselno. Zato lahko trdimo, da pomožna hipoteza 2, „znanje, pridobljeno med srednješolskim izobraževanjem, nima statistično pomembnega vpliva na razlike med uspešnostjo učencev v eksperimentalni skupini“, ni bilo potrjeno.

Raziskava, ki je bila opisana v okviru te disertacije, je naletela na številne težave tako organizacijske kot tehnične narave, ki jih je treba upoštevati pri preučevanju rezultatov raziskave in jih primerjati z rezultati prejšnjih podobnih raziskav. Raziskave, ki vključujejo določen eksperimentalni program, ki se izvaja v daljšem časovnem obdobju, je v praksi zapleteno organizirati, ker morajo udeleženci v tem opaziti korist, poleg tega pa je potrebno zagotoviti, da se program prilagodi okvirom predmeta pri katerem gostujejo. Upoštevati je potrebno tudi tehnične podrobnosti, na primer učna sredstva, ki so na voljo v učilnici, v kateri se program izvaja, to so bili v tem primeru računalniki, na katerih je nameščen GeoGebra 5, in gradivo, ki ga je zagotovil raziskovalec, ki je vodil program, kot so prosojnice, leseni modeli, s katerimi so študenti delali, itd. Ena izmed glavnih skrbi raziskovalcev, ki izvajajo tovrstno raziskovanje učinkov daljših programov, je, koliko študentov se je udeležilo vseh lekcij in koliko jih je preskočilo katero od njih. V raziskavah, kjer končno testiranje meri učinek celotnega eksperimentalnega programa, preiskovancev, ki se niso udeležili vseh lekcij, ni mogoče upoštevati in jih je treba izključiti iz vzorca, kar pa je mogoče ugotoviti le na koncu raziskave.

Vaje, razvite in uporabljene v eksperimentalnem programu, so bile dobro sprejete, predstavljeni matematični problemi so bili na ustrezni stopnji zahtevnosti, GeoGebra pa je sama ponudila orodja, ki so študentom omogočala razumevanje problemov, ki so jih imeli pri predhodnih nalogah, kot tudi tudi učenje iz izkušenj. Orodje za vrtenje se je izkazalo za zelo uporabno, pa tudi različni načini modeliranja, ki jih programska oprema ponuja, tako da je lahko vsak posameznik izbral, katero metodo bo uporabil za reševanje nalog glede na svoje želje in vzgibe. Stopnjo zahtevnosti predstavljenih nalog je mogoče enostavno prilagoditi z dodajanjem bolj zapletenih problemov, če preiskovanci pokažejo več sposobnosti pri reševanju in odstranjevanjem problemov iz nalog, če imajo preiskovanci težave pri reševanju bolj zapletenih nalog, kar pa na splošno ne bi smelo vplivati na učinek eksperimentalnega programa.

Ključne besede: *eksperimentalni program, GeoGebra, izboljšanje prostorskih sposobnosti, prostorske sposobnosti*.