

### UČNI NAČRT PREDMETA / COURSE SYLLABUS

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|---------------|------------------------|
| Predmet:      | Diferencialne enačbe   |
| Course title: | Differential equations |

| Študijski program in stopnja<br>Study programme and level | Študijska smer<br>Study field | Letnik<br>Academic year | Semester<br>Semester |
|---|-------------------------------|-------------------------|----------------------|
| Matematika, 3. stopnja                                    |                               | 1.                      | 2.                   |
| Mathematics, 3 <sup>rd</sup> Degree                       |                               | 1 <sup>st</sup>         | 2 <sup>nd</sup>      |

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| Vrsta predmeta / Course type | obvezni ali izbirni/obligatory or elective |
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| Univerzitetna koda predmeta / University course code: |  |
|---|--|

| Predavanja<br>Lectures | Seminar<br>Seminar | Vaje<br>Tutorial | Klinične vaje<br>work | Druge oblike<br>študija | Samost. delo<br>Individ.<br>work | ECTS |
|------------------------|--------------------|------------------|-----------------------|-------------------------|----------------------------------|------|
| 45                     |                    |                  |                       |                         | 225                              | 9    |

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| Nosilec predmeta / Lecturer: | Valerij Romanovskij |
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| Jeziki /<br>Languages: | Predavanja /<br>Lectures:<br>Vaje / Tutorial: | Slovenski / Slovene<br>Slovenski / Slovene |
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**Pogoji za vključitev v delo oz. za opravljanje  
študijskih obveznosti:**

Znanje osnovnih pojmov in rezultatov iz teorije NDE

Basic knowledge of fundamental notions and results of  
the theory of ODE's

**Vsebina:**

- diferencialne enačbe 2. reda
- približne rešitve linearnih diferencialnih enačb
- približne rešitve nelinearnih diferencialnih enačb
- reguarna in singularna perturbacijska teorija
- perturbacijske metode za probleme lastnih vrednosti
- aproksimacije WKB
- tokovi in invariantni podprostori
- funkcije Ljapunova
- normalne forme diferencialnih enačb in preslikav
- bifurkacije ravnovesne lege
- bifurkacije periodičnih orbit
- izohronost nihanj
- uvod v kaos

**Content (Syllabus outline):**

- second order ODEs
- approximate solutions of linear differential equations
- approximate solutions of nonlinear differential equations
- regular and singular perturbation theory
- perturbations methods for the eigenvalues problem
- WKB approximations
- flows and invariant subspaces
- Lyapunov functions
- normal forms of differential equations and maps
- bifurcations of singular points
- bifurcations of periodic orbits
- isochronicity of oscillations
- an introduction to chaos

**Temeljni literatura in viri / Readings:**

- D.K. Arrowsmith, C.M. Place, Dynamical systems. Differential equations, maps and chaotic behaviour, Chapman and Hall Mathematics Series, Chapman & Hall, London 1992.
- C. M. Bender, S. A. Orszag, Advanced mathematical methods for scientists and engineers, International series in pure and applied mathematics, McGraw-Hill Book Co., New York 1978.
- S. N. Chow, J. K. Hale, Methods of bifurcation theory, Grundlehren der Mathematischen wissenschaften, 251. Springer-Verlag, New York – Berlin 1982.
- J. Guckenheimer, P. Holmes, Nonlinear oscillations, dynamical systems and bifurcations of vector fields, Applied Mathematical sciences, 42, Springer-Verlag, New York 1983.
- J. A. Murdock, Normal forms and unfoldings for local dynamical systems, Springer, New York, 2003
- V. G. Romanovski, D.S. Shafer, The Center and Cyclicity Problems A Computational Algebra Approach. Birkhäuser, Boston, 2009

**Cilji in kompetence:**

- Razumevanje osnovnih načinov kvalitativne in bifurkacijske analize diferencialnih enačb
- Poznavanje metod študija lastnosti rešitev diferencialnih enačb in gladkih preslikav
- Pridobiti si sposobnost detajlne analize določenih matematičnih modelov opisanih z navadnimi diferencialnimi enačbami ali gladkimi preslikavami
- Razviti sposobnost samostojnega razvijanja novega znanja s področja diferencialnih enačb
- Zmožnost razvijanja kritične refleksije na področju diferencialnih enačb
- Razviti zmožnost vodenja najzahtevnejših znanstvenoraziskovalnih projektov s širšega področja diferencialnih enačb.

**Objectives and competences:**

- Understanding main approaches to the qualitative and bifurcational analysis of differential equations
- Gaining knowledge of methods of studying the properties of solutions of differential equations and smooth maps
- Gaining skills of detail analysis of certain mathematical model described by ordinary differential equations or smooth maps
- To develop the ability to independently develop new knowledge in the field of differential equations
- Ability to develop critical reflection in differential equations
- To develop the ability to lead the most challenging scientific research projects in the wider field of differential equations

**Predvideni študijski rezultati:****Intended learning outcomes:**

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| <p><u>Znanje in razumevanje:</u></p> <ul style="list-style-type: none"> <li>• Razumevanje metod kvalitativne in bifurkacijske analize dinamičnih sistemov</li> <li>• Pridobivanje sposobnosti sistematskega študija rešitev dinamičnih sistemov in njihovih lastnosti.</li> <li>• Sposobnost uporabe znanja za študij matematičnih modelov različnih procesov in pojavov v fizikalni, tehnični in drugih uporabnih znanosti</li> <li>• Sposobnost razumevanja in analiziranja dinamičnih procesov opisanih diferencialnimi enačbami in gladkimi preslikavami</li> </ul> | <p><u>Knowledge and understanding:</u></p> <ul style="list-style-type: none"> <li>• Understanding of methods of qualitative and bifurcational analysis of dynamical systems</li> <li>• Gaining some systematic approaches to studying of solutions of dynamical systems and their properties</li> <li>• The ability to use of knowledge for studying of mathematical models of various processes and phenomena arising in physical, technical and other applied sciences</li> <li>• The ability to understand and analyze the dynamics of processes described by differential equations and smooth maps</li> </ul> |
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### Metode poučevanja in učenja:

- predavanja;
- priprava seminarja;
- konzultacije;
- samostojni študij.

### Learning and teaching methods:

- lectures;
- seminar work;
- consultations;
- self-study.

Delež (v %) /

### Načini ocenjevanja:

Weight (in %)

### Assessment:

| Način (pisni izpit, ustno izpraševanje, naloge, projekt): | Weight (in %) | Type (examination, oral, coursework, project): |
|---|---------------|--|
| • seminarsko predavanje;                                  | <b>20%</b>    | • seminar talk;                                |
| • pisni izpit;  | <b>30%</b>    | • written work;                                |
| • ustno izpraševanje.                                     | <b>50%</b>    | • oral examination.                            |

### Reference nosilca / Lecturer's references:

1. ROMANOVSKI, Valery, FERNANDES, Wilker, OLIVEIRA, Regilene. Bi-center problem for some classes of [Z] [sub] 2-equivariant systems. *Journal of Computational and Applied Mathematics*, ISSN 0377-0427. [Print ed.], 2017, vol. 320, str. 61-75, doi: [10.1016/j.cam.2017.02.003](https://doi.org/10.1016/j.cam.2017.02.003). [COBISS.SI-ID [23085576](#)]
2. FERNANDES, Wilker, ROMANOVSKI, Valery, SULTANOVA, Marzhan, TANG, Yilei. Isochronicity and linearizability of a planar cubic system. *Journal of mathematical analysis and applications*, ISSN 0022-247X. [Print ed.], 2017, vol. 450, iss. 1, str. 795-813, doi: [10.1016/j.jmaa.2017.01.058](https://doi.org/10.1016/j.jmaa.2017.01.058). [COBISS.SI-ID [22987784](#)]
3. ROMANOVSKI, Valery, SHAFER, Douglas. Complete integrability and time-reversibility of some 3-dim systems. *Applied Mathematics Letters*, ISSN 0893-9659. [Print ed.], January 2016, vol. 51, str. 27-33, doi: [10.1016/j.aml.2015.07.006](https://doi.org/10.1016/j.aml.2015.07.006). [COBISS.SI-ID [21562120](#)]
4. SHENG, Lijuan, HAN, Maoan, ROMANOVSKI, Valery. On the number of limit cycles by perturbing a piecewise smooth Liénard model. *International journal of bifurcation and chaos in applied sciences and engineering*, ISSN 0218-1274, 2016, vol. 26, no. 10, str. 1650168-1-1650168-16, doi: [10.1142/S0218127416501686](https://doi.org/10.1142/S0218127416501686). [COBISS.SI-ID [23084040](#)]
5. DU, Zengji, ROMANOVSKI, Valery, ZHANG, Xiang. Varieties and analytic normalizations of partially integrable systems. *Journal of differential equations*, ISSN 0022-0396, 2016, vol. 260, iss. 9, str. 6855-6871, doi: [10.1016/j.jde.2016.01.009](https://doi.org/10.1016/j.jde.2016.01.009). [COBISS.SI-ID [22043144](#)]