Univerza v Mariboru

Fakulteta za naravoslovje in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	**Teorija mere**
Course title:	**Measure Theory**

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika, 2. stopnja	Modul S1	**1. ali 2.**	**1. ali 3.**
Mathematics, 2nd cycle	Module S1	**1. or 2.**	**1. or 3.**

Vrsta predmeta / Course type	obvezni / compulsory
Univerzitetna koda predmeta / University course code:	

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
60		**45**			**165**	**9**

Nosilec predmeta / Lecturer:	Valerij Romanovskij

Jeziki / Languages:	**Predavanja / Lectures:**	SLOVENSKO/SLOVENE
	Vaje / Tutorial:	SLOVENSKO/SLOVENE

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	**Prerequisits:**

Vsebina:	**Content (Syllabus outline):**
• Osnovni pojmi teorije mere: Algebra, σ-algebra, Borelova σ-algebra na R^n. Mere in osnovne lastnosti mer. Merljivi prostori. Pozitivne mere. Zunanje mere. Lebesqueova mera na R^n. • Funkcije in integrali: Merljive funkcije. Stopničaste funkcije. Integral stopničaste funkcije. Integral merljive funkcije. Izrek o monotoni konvergenci. Fatoujeva lema in Lebesqueov izrek o dominantni	• Basic concepts of measure theory: Algebra, σ-algebra, Borel σ-algebra on R^n. Measure and its basic properties. Measurable spaces. Positive measures. Outer measures. Lebesque measure on R^n. • Functions and integrals: Measurable functions. Simple measurable functions. The integral of a simple measurable function. The integral of a measurable function. The monotone convergence theorem. Fatou's

konvergenci. Povezanost Riemannovega in Lebesqueovega integrala. • Konvergenca: Zaporedja merljivih funkcij in konvergenca. Konvergenca skoraj povsod. Norma in normirani L^p-prostori. Neenakosti (Hölder, Minkowski). Dualni prostori. • Predznačne in kompleksne mere: Predznačne mere in Hahnov razcepni izrek. Kompleksne mere in Radon-Nikodymov izrek. Funkcije z omejeno varianco. • Produktne mere: Merjenje in integriranje po produktnih prostorih (Fubinijev izrek). • Odvajanje: Odvodi mer. Odvodi funkcij. • Rieszov izrek o reprezentaciji pozitivnih linearnih funkcionalov na C(X). • Lebesgue-Stieltjesov integral.	lemma and Lebesque's dominated convergence theorem. Relationships between Riemann's and Lebesque's integral. • Convergence: Sequences of measurable functions and convergence. Convergence almost everywhere. Norm and normed L^p-spaces. Inequalities (Hölder, Minkowski). Dual spaces. • Signed and complex measures: Signed measures and the Hahn decomposition theorem. Complex measures and the Radon-Nikodym theorem. Functions of bounded variation. • Product measures: Measures and integrals on product spaces (Fubini's theorem). • Differentiation: Differentiation of measures. Differentiation of functions. • The Riesz representation theorem on positive linear functionals on C(X). • Lebesgue-Stieltjes integral

Temeljni literatura in viri / Readings:

1. M. Capinski, E. Kopp: *Measure, integral and probability*, Springer-Verlag London, 2004.
2. D. L. Cohn: *Measure theory,* Birkhäuser, 1994.
3. R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA, 2001.
4. M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA, 1985.
5. W. Rudin: *Real and complex analysis*, *3th edition*, Mc-Graw-Hill, 1986.
6. H. Sohrab, *Basic real analysis*, Birkhauser Boston, 2003.
7. I. Vidav, *Višja matematika II*, DZS, Ljubljana, 1975.

Cilji in kompetence:	**Objectives and competences:**
Glavni cilj predmeta je proučiti temeljne koncepte in rezultate teorije mere.	The main goal of the course is to study the fundamental concepts and results of measure theory.

Predvideni študijski rezultati:	**Intended learning outcomes:**
Znanje in razumevanje: • merljivi prostori, merljive funkcije, abstraktno integriranje, izreki o konvergenci, L^p-prostori, produktne mere, odvodi mer. Prenesljive/ključne spretnosti in drugi atributi: • Poznavanje osnov teorije mere je podlaga za študij različnih matematičnih področij (funkcionalne	Knowledge and Understanding: • Measurable spaces, measurable functions, abstract integration, convergence theorems, L^p-spaces, product measures, differentiation of measures. Transferable/Key Skills and other attributes: • Knowing the fundamentals of measure theory is a prerequisite for studying various mathematical areas (functional analysis,

analize, verjetnosti, parcialnih diferencialnih enačb itd.).	probability, partial differential equations etc.).

Metode poučevanja in učenja:	**Learning and teaching methods:**
• Predavanja • Teoretične vaje	• Lectures • Theoretical exercises

Načini ocenjevanja: / **Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):	Delež (v %) / Weight (in %)	Type (examination, oral, coursework, project):
• Pisni izpit – problemi • Pisni izpit – teorija	50% 50%	• Written exam – problems • Written exam – theory
Pisni izpit - problemi se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) Pisni izpit - teorja se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.		Written exam – problems can be replaced by two parital tests (mid-term testing) Written exam – theory can be replaced by two parital tests (mid-term testing) Each of the mentioned commitments must be assessed with a passing grade.

Reference nosilca / Lecturer's references:

1. FERČEC, Brigita, ROMANOVSKI, Valery, TANG, Yilei, ZHANG, Ling. Integrability and bifurcation of a three-dimensional circuit differential system. *Discrete and continuous dynamical systems. Series B.* 2022, vol. 27, iss. 8, str. 4573-4588. ISSN 1531-3492. DOI: 10.3934/dcdsb.2021243. [COBISS.SI-ID 88277507]

2. ARCET, Barbara, ROMANOVSKI, Valery. Integrability and linearizability of symmetric three-dimensional quadratic systems. *Discrete and continuous dynamical systems. Series S*. April 2022, 18 str. ISSN 1937-1632. DOI: 10.3934/dcdss.2022104. [COBISS.SI-ID 130109955]

3. WANG, Qinlong, YU'E, Xiong, HUANG, Wentao, ROMANOVSKI, Valery. Isolated periodic wave trains in a generalized Burgers–Huxley equation. *Electronic journal of qualitative theory of differential equations*. 2022, vol. 2022, no. 4, 16 str. ISSN 1417-3875. http://www.math.u-szeged.hu/ejqtde/p9524.pdf, DOI: 10.14232/ejqtde.2022.1.4. [COBISS.SI-ID 110159107]

4. ARCET, Barbara, GINÉ, Jaume, ROMANOVSKI, Valery. Linearizability of planar polynomial Hamiltonian systems. *Nonlinear analysis: real world applications*. Feb. 2022, vol. 63, 19 str. ISSN 1468-1218. DOI: 10.1016/j.nonrwa.2021.103422. [COBISS.SI-ID 110154755]

5. LI, Yongjun, ROMANOVSKI, Valery. Isochronous solutions of a 3-dim symmetric quadratic system. *Applied mathematics and computation*. [Print ed.]. 15 Sept. 2021, vol. 405, 12 str. ISSN

0096-3003. DOI: 10.1016/j.amc.2021.126250. [COBISS.SI-ID 95936003]