Univerza v Mariboru

Fakulteta za naravoslovje in matematiko

| UČNI NAČRT PREDMETA / COURSE SYLLABUS | |
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| **Predmet:** | **Teorija mere** |
| **Course title:** | **Measure Theory** |

| Študijski program in stopnja<br>Study programme and level | Študijska smer<br>Study field | Letnik<br>Academic year | Semester<br>Semester |
|---|---|---|---|
| Matematika, 2. stopnja | Modul F1 | **1. ali 2.** | **1. ali 3.** |
| Mathematics, 2nd cycle | Module F1 | **1. or 2.** | **1. or 3.** |

**Vrsta predmeta / Course type** obvezni/compulsory

**Univerzitetna koda predmeta / University course code:**

| Predavanja<br>Lectures | Seminar<br>Seminar | Sem. vaje<br>Tutorial | Lab. vaje<br>Laboratory work | Teren. vaje<br>Field work | Samost. delo<br>Individ. work | ECTS |
|---|---|---|---|---|---|---|
| **60** | | **45** | | | **165** | **9** |

**Nosilec predmeta / Lecturer:** Valerij Romanovskij

| **Jeziki / Languages:** | **Predavanja / Lectures:** | SLOVENSKO/SLOVENE |
|---|---|---|
| | **Vaje / Tutorial:** | SLOVENSKO/SLOVENE |

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisits:**

**Vsebina:**

- Osnovni pojmi teorije mere: Algebra, σ-algebra, Borelova σ-algebra na $R^n$. Mere in osnovne lastnosti mer. Merljivi prostori. Pozitivne mere. Zunanje mere. Lebesqueova mera na $R^n$.
- Funkcije in integrali: Merljive funkcije. Stopničaste funkcije. Integral stopničaste funkcije. Integral merljive funkcije. Izrek o monotoni konvergenci. Fatoujeva lema in Lebesqueov izrek o dominantni

**Content (Syllabus outline):**

- Basic concepts of measure theory: Algebra, σ-algebra, Borel σ-algebra on $R^n$. Measure and its basic properties. Measurable spaces. Positive measures. Outer measures. Lebesque measure on $R^n$.
- Functions and integrals: Measurable functions. Simple measurable functions. The integral of a simple measurable function. The integral of a measurable function. The monotone convergence theorem. Fatou's

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| konvergenci. Povezanost Riemannovega in Lebesqueovega integrala.<br>• Konvergenca: Zaporedja merljivih funkcij in konvergenca. Konvergenca skoraj povsod. Norma in normirani $L^p$-prostori. Neenakosti (Hölder, Minkowski). Dualni prostori.<br>• Predznačne in kompleksne mere: Predznačne mere in Hahnov razcepni izrek. Kompleksne mere in Radon-Nikodymov izrek. Funkcije z omejeno varianco.<br>• Produktne mere: Merjenje in integriranje po produktnih prostorih (Fubinijev izrek).<br>• Odvajanje: Odvodi mer. Odvodi funkcij.<br>• Rieszov izrek o reprezentaciji pozitivnih linearnih funkcionalov na C(X).<br>• Lebesgue-Stieltjesov integral. | lemma and Lebesque's dominated convergence theorem. Relationships between Riemann's and Lebesque's integral.<br>• Convergence: Sequences of measurable functions and convergence. Convergence almost everywhere. Norm and normed $L^p$-spaces. Inequalities (Hölder, Minkowski). Dual spaces.<br>• Signed and complex measures: Signed measures and the Hahn decomposition theorem. Complex measures and the Radon-Nikodym theorem. Functions of bounded variation.<br>• Product measures: Measures and integrals on product spaces (Fubini's theorem).<br>• Differentiation: Differentiation of measures. Differentiation of functions.<br>• The Riesz representation theorem on positive linear functionals on C(X).<br>• Lebesgue-Stieltjes integral |

**Temeljni literatura in viri / Readings:**


1. M. Capinski, E. Kopp: *Measure, integral and probability*, Springer-Verlag London, 2004.
2. D. L. Cohn: *Measure theory,* Birkhäuser, 1994.
3. R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA, 2001.
4. M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA, 1985.
5. W. Rudin: *Real and complex analysis*, *3th edition*, Mc-Graw-Hill, 1986.
6. H. Sohrab, *Basic real analysis*, Birkhauser Boston, 2003.
7. I. Vidav, *Višja matematika II*, DZS, Ljubljana, 1975.


| **Cilji in kompetence:** | **Objectives and competences:** |
|---|---|
| Glavni cilj predmeta je proučiti temeljne koncepte in rezultate teorije mere. | The main goal of the course is to study the fundamental concepts and results of measure theory. |

| **Predvideni študijski rezultati:** | **Intended learning outcomes:** |
|---|---|
| Znanje in razumevanje:<br>• merljivi prostori, merljive funkcije, abstraktno integriranje, izreki o konvergenci, $L^p$-prostori, produktne mere, odvodi mer.<br><br>Prenesljive/ključne spretnosti in drugi atributi:<br>• Poznavanje osnov teorije mere je podlaga za študij različnih | Knowledge and Understanding:<br>• Measurable spaces, measurable functions, abstract integration, convergence theorems, $L^p$-spaces, product measures, differentiation of measures.<br><br>Transferable/Key Skills and other attributes:<br>• Knowing the fundamentals of measure theory is a prerequisite for studying various |

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| matematičnih področij (funkcionalne analize, verjetnosti, parcialnih diferencialnih enačb itd.). | mathematical areas (functional analysis, probability, partial differential equations etc.). |

| **Metode poučevanja in učenja:** | **Learning and teaching methods:** |
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| • Predavanja<br>• Teoretične vaje | • Lectures<br>• Theoretical exercises |

**Načini ocenjevanja:** / **Assessment:**

| Način (pisni izpit, ustno izpraševanje, naloge, projekt): | Delež (v %) / Weight (in %) | Type (examination, oral, coursework, project): |
|---|---|---|
| • Pisni izpit – problemi<br>• Pisni izpit – teorija | 50%<br>50% | • Written exam – problems<br>• Written exam – theory |
| Pisni izpit - problemi se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) | | Written exam – problems can be replaced by two parital tests (mid-term testing) |
| Pisni izpit - teorja se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) | | Written exam – theory can be replaced by two parital tests (mid-term testing) |
| Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno. | | Each of the mentioned commitments must be assessed with a passing grade. |

**Reference nosilca / Lecturer's references:**


1. XIA, Yong-Hui, GRAŠIČ, Mateja, HUANG, Wentao, ROMANOVSKI, Valery. Limit cycles in a model of olfactory sensory neurons. International journal of bifurcation and chaos in applied sciences and engineering, ISSN 0218-1274, 2019, vol. 29, no. 3, str. 1950038-1-1950038-9.

2. ANTONOV, Valery, FERNANDES, Wilker, ROMANOVSKI, Valery, SHCHEGLOVA, Natalie L. Firstintegrals of the May-Leonard asymmetric system. Mathematics, ISSN 2227-7390, 2019, vol. 7, no. 3, str. 1-15.

3. DUKARIĆ, Maša, ERRAMI, Hassan, JERALA, Roman, LEBAR, Tina, ROMANOVSKI, Valery,TÓTH, János, WEBER, Andreas. On three genetic repressilator topologies. Reaction kinetics, mechanisms and catalysis, ISSN 1878-5190. [Print ed.], Feb. 2019, vol. 126, iss. 1, str. 3-30.

4. HAN, Maoan, PETEK, Tatjana, ROMANOVSKI, Valery. Reversibility in polynomial systems of ODE's.Applied mathematics and computation, ISSN 0096-3003. [Print ed.], 2018, vol. 338, str. 55-71.

5. LIANG, Feng, ROMANOVSKI, Valery, ZHANG, Daoxiang. Limit cycles in small perturbations of a planar piecewise linear Hamiltonian system with a non-regular separation line : an introduction. Chaos, solitons and fractals. [Print ed.], June 2018, vol. 111, str. 18-34.