



Univerza v Mariboru

Fakulteta za naravoslovje
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS

Predmet: Poglavja iz topologije
Course title: Topics in Topology

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Izobraževalna matematika, 2. stopnja		1. ali 2.	1. ali 3.
Educational mathematics, 2 nd cycle		1. or 2.	1. or 3.

Vrsta predmeta / Course type

izbirni/elective

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
45	15				120	6

Nosilec predmeta / Lecturer:

Iztok BANIČ

**Jeziki /
Languages:**

**Predavanja /
Lectures:** SLOVENSKO/SLOVENIAN

Vaje / Tutorial: SLOVENSKO/SLOVENIAN

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Poznavanje osnov splošne topologije.

Vsaka izmed naštetih obveznosti v načinih ocenjevanja mora biti opravljena s pozitivno oceno.

Pozitivni oceni pri ustni predstavitvi in sprotih nalogah sta pogoj za pristop k ustnemu izpitu.

Prerequisites:

Knowledge of introduction into general topology.

Each of the mentioned commitments must be assessed with a passing grade.

Passing grades of the oral presentation and the coursework are required for taking the oral exam.

Vsebina:

Vsebina predmeta se prilagaja aktualnim potrebam in razvoju.

1. Metrizabilnost:

- Metrizabilni prostori; omejene metrike, ki dajo isto topologijo; topološki produkti števno mnogo metrizabilnih prostorov,
- Urysonohnova lema; Tietzejev razširitveni izrek.
- Urysohnov izrek o metrizabilnosti.

2. Teorije kontinuumov:

- Kontinuumi. Zgledi kontinuumov. Vgnezdjeni preseki. Verige.
- Osnovne lastnosti.
- Kompozanti.
- Posebni primeri kontinuumov. Knasterjev kontinuum, psevdolok, pahljače, grafi.
- Krivulje, ki zapolnijo prostor.
- Izreki o napihovanju roba množice.
- Lastnost fiksne točke.
- Peanovi kontinuumi.

3. Hiperprostori:

- Hausdorffova metrika in hiperprostori.
- Limsup, liminf, lim.
- Izreki o konvergenzi množic.

4. Inverzna zaporedja in inverzne limite:

- Inverzne limite kontinuumov;
- Nerazcepnost inverznih limit
- Anderson-Choquetov vložitveni izrek
- Kontinuumi, ki so podobni loku (drevesom)
- Mahavierjevi sistemi.

Content (Syllabus outline):

The contents of this subject is adjusted to the current needs and development.

1. Metrizability:

- Metrizable spaces; bonded metrics that give the same topology; topological products of countably many metrizable spaces.
- The Urysohn lemma; the Tietze extension theorem.
- The Urysohn Metrization Theorem

2. Continuum theory:

- Continua. Examples of continua. Nested intersections. Chains.
- Basic properties
- Composants.
- Special examples. Knaster continuum, pseudoarc, fans, graphs.
- Space filling curves.
- Boundary bumping theorems.
- Fixed-point property.
- Peano continua.

3. Hyperspaces.

- Hausdorff metric and hyperspaces
- Limsup, liminf, lim.
- Convergence of sets theorems.

4. Inverse sequences and inverse limits:

- Inverse limits of continua
- Indecomposability of inverse limits.
- Anderson-Choquet embedding theorem.
- Arc-like (tree-like) continua.
- Mahavier systems.

Temeljni literatura in viri / Readings:

S .B. Nadler: Continuum theory: an introduction, Marcel Dekker, New York, 1992
A. Illanes, S. B. Nadler: Hyperspaces. Fundamentals and recent advances, Marcel Dekker, Inc., New York, 1999
J.R.Munkres: Topology: a first course, Englewood Cliffs, NJ, Prentice-Hall, 1975

Dodatna literatura/Additional Readings:

W.T. Ingram, W. S. Mahavier: Inverse limits: from continua to chaos, Springer, New York, 2012.

Cilji in kompetence:

Cilj in kompetence tega predmeta so, da študentje usvojijo pojme in metode poglavij iz topologije (temeljito spoznati in uporabljati klasične izreke metrizabilnih prostorov; kontinuum in njihove lastnosti; hiperprostore in njihove lastnosti; inverzna zaporedja in inverzne limite kontinuumov) in jih spretno uporabljajo pri raziskovalnem delu in nadaljnjem študiju topologije.

Objectives and competences:

The objectives and competences of this course are for students to acquire knowledge of topics in topology (to know thoroughly classical theorems of metrizable spaces; continua and their properties; hyperspaces and their properties; inverse sequences and inverse limits of continua), and to thoroughly apply it in the further study of topology.

Predvideni študijski rezultati:

Znanje in razumevanje:

Po zaključku tega predmeta bo študent sposoben

- razumeti pojme poglavij iz topologije.
- razložiti in uporabljati izreke poglavij iz topologije.
- za reševanje problemov uporabiti izkušnje iz izbranih poglavij topologije.
- razumeti in uporabljati klasične izreke metrizabilnih prostorov.
- razumeti in uporabljati lastnosti kontinuumov.
- razumeti in uporabljati konstrukcijske metode za konstrukcijo novih primerov kontinuumov.
- razumeti in uporabljati lastnosti hiperprostorov in z njihovo pomočjo konstruirati nove prostore.
- razumeti in uporabljati inverzne limite inverznih zaporedij in z njimi graditi nove prostore.

Intended learning outcomes:

Knowledge and understanding:

On completion of this course the student will be able to

- understand concepts of topics in topology.
- explain and use theorems from selected topics of topology,
- apply experience from selected topics in topology for problem solving.
- understand concepts of classical theorems of metrizable spaces and know their applications.
- be able to understand different properties of continua.
- be able to understand and implement construction methods for constructions of new examples of continua.
- be able to understand and implement construction of hyperspaces.
- be able to understand and implement construction of spaces using inverse systems.

Prenosljive/ključne spretnosti in drugi atributi: • Spretnosti komuniciranja: ustni zagovor izpita, pisno izražanje pri pisnem izpitu. • Uporaba informacijske tehnologije: uporaba računalna ali računalniških aplikacij pri reševanju problemov. • Reševanje problemov: reševanje problemov s pomočjo metod iz izbranih poglavij iz topologije. • Uporaba znanja pri samostojnem raziskovalnem delu. • Prenos znanja obravnavanih metod na druga področja, predvsem na področja analize, kompleksne analize, teorije grafov, geometrije in topologije.	Transferable/Key skills and other attributes: • Communication skills: oral exam, manner of expression at written examination. • Use of information technology: use of a calculator or computer applications for problem solving. • Problem solving: problem solving using methods from selected topics in topology. • To apply the knowledge and skills to individual research work. • Knowledge transfer of treated methods into other fields, to analysis, complex analysis, graph theory, geometry and topology.						
Metode poučevanja in učenja: • Predavanja • Seminar • Individualno delo	Learning and teaching methods: • Lectures • Seminar • Individual work						
Načini ocenjevanja:	Assessment:						
Ustni izpit Ustna predstavitev Sprotne naloge	<table> <tr> <td>50%</td><td>Oral exam</td></tr> <tr> <td>25%</td><td>Oral presentation</td></tr> <tr> <td>25%</td><td>Coursework</td></tr> </table>	50%	Oral exam	25%	Oral presentation	25%	Coursework
50%	Oral exam						
25%	Oral presentation						
25%	Coursework						

Reference nosilca / Lecturer's references:

1. BANIČ, Iztok, ERCEG, Goran, KENNEDY, Judy A., MOURON, Christopher, NALL, Van. Transitive mappings on the Cantor fan. Ergodic theory & dynamical systems. 2025, 35 str. ISSN 0143-3857. <https://doi.org/10.1017/etds.2025.6>, DOI: 10.1017/etds.2025.6.
2. BANIČ, Iztok, ERCEG, Goran, KENNEDY, Judy A., MOURON, Christopher, NALL, Van. Chaos and mixing homeomorphisms on fans. Journal of difference equations and applications. 2025, vol. 31, no. 1, 31 str. ISSN 1023-6198. Digitalna knjižnica Univerze v Mariboru – DKUM, DOI: 10.1080/10236198.2024.2384947, DOI: 20.500.12556/DKUM-90072.
3. BANIČ, Iztok, ERCEG, Goran, KENNEDY, Judy A. An embedding of the Cantor fan into the Lelek fan. Rad Hrvatske akademije znanosti i umjetnosti. Razred za matematičke, fizičke i kemijske znanosti. Matematičke znanosti. 2025, vol. 29 = 564, str. 221-229. ISSN 1845-4100. <https://doi.org/10.21857/m8vqrt3lk9>, Digitalna knjižnica Univerze v Mariboru – DKUM, DOI: 10.21857/m8vqrt3lk9, DOI: 20.500.12556/DKUM-91938.

4. BANIČ, Iztok, GRIL ROGINA, Rene, KENNEDY, Judy A., NALL, Van. Sufficient conditions for non-zero entropy of closed relations. *Ergodic theory & dynamical systems*. Nov. 2024, vol. 44, iss. 11, str. 3091-3119. ISSN 0143-3857. DOI: 10.1017/etds.2024.11.

5. BANIČ, Iztok, ČREPNIJAK, Matevž, KAC, Teja. Markov set-valued functions on compact metric spaces. *Glasnik matematički*. Serija 3. 2024, vol. 59, no. 1, str. 193-212. ISSN 0017-095X. [https://web.math.pmf.unizg.hr/glasnik/59.1/59\(1\)-09.pdf](https://web.math.pmf.unizg.hr/glasnik/59.1/59(1)-09.pdf), DOI: 10.3336/gm.59.1.09.