Univerza v Mariboru

Fakulteta za naravoslovje in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	**Osnove teorije mere**
Course title:	**The Basics of Measure Theory**

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Izobraževalna matematika – enopredmetna, 2. stopnja		**2**	**3**
Educational mathematics single-major, 2nd cycle		**2**	**3**

Vrsta predmeta / Course type izbirni/elective

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
45		**30**			**105**	**6**

Nosilec predmeta / Lecturer: Valerij Romanovskij

Jeziki / Languages:		
	Predavanja / Lectures:	SLOVENSKO/SLOVENE
	Vaje / Tutorial:	SLOVENSKO/SLOVENE

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Prerequisits:

Vsebina:

- Osnovni pojmi teorije mere: Algebra, σ-algebra, Borelova σ-algebra na R^n. Mere in osnovne lastnosti mer. Merljivi prostori. Pozitivne mere. Zunanje mere. Lebesgueova mera na R^n.

Content (Syllabus outline):

- Basic concepts of measure theory: Algebra, σ-algebra, Borel σ-algebra on R^n. Measure and its basic properties. Measurable spaces. Positive measures. Outer measures. Lebesgue measure on R^n.

- Funkcije in integrali: Merljive funkcije. Stopničaste funkcije. Integral stopničaste funkcije. Integral merljive funkcije. Izrek o monotoni konvergenci. Fatoujeva lema in Lebesgueov izrek o dominantni konvergenci. Povezanost Riemannovega in Lebesgueovega integrala.
- Konvergenca: Zaporedja merljivih funkcij in konvergenca. Konvergenca skoraj povsod. Norma in normirani L^p-prostori. Neenakosti (Hölder, Minkowski). Dualni prostori.
- Predznačne in kompleksne mere: Predznačne mere in Hahnov razcepni izrek. Kompleksne mere in Radon-Nikodymov izrek. Funkcije z omejeno varianco.

- Functions and integrals: Measurable functions. Simple measurable functions. The integral of a simple measurable function. The integral of a measurable function. The monotone convergence theorem. Fatou's lemma and Lebesgue's dominated convergence theorem. Relationships between Riemann's and Lebesgue's integral.
- Convergence: Sequences of measurable functions and convergence. Convergence almost everywhere. Norm and normed L^p-spaces. Inequalities (Hölder, Minkowski). Dual spaces.
- Signed and complex measures: Signed measures and the Hahn decomposition theorem. Complex measures and the Radon-Nikodym theorem. Functions of bounded variation.

Temeljni literatura in viri / Readings:

1. M. Capinski, E. Kopp: *Measure, integral and probability*, Springer-Verlag London, 2004.
2. D. L. Cohn: *Measure theory,* Birkhäuser, 1994.
3. R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA, 2001.
4. M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA, 1985.
5. W. Rudin: *Real and complex analysis*, *3th edition*, Mc-Graw-Hill, 1986.
6. H. Sohrab, *Basic real analysis*, Birkhauser Boston, 2003.
7. I. Vidav, *Višja matematika II*, DZS, Ljubljana, 1975.

Cilji in kompetence:

Glavni cilj predmeta je proučiti temeljne koncepte in rezultate teorije mere.

Objectives and competences:

The main goal of the course is to study the fundamental concepts and results of measure theory.

Predvideni študijski rezultati:

Znanje in razumevanje:

Po zaključku tega predmeta bo študent sposoben
- Prepoznati in uporabiti glavne pojime teorije mere,
- izbrati ustrezno metodo za izračun mere in Lebesguejevega integrala,
- analizirati preproste tipe problemov iz naravoslovja in tehnike z uporabo teorije mere.
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Prenosljive/ključne spretnosti in drugi atributi:

- *Spretnosti komuniciranja:* nedvoumno in natančno izražanje, pisno in ustno.
- *Spretnosti računanja:* učenje raznovrstnih računskih postopkov.

Intended learning outcomes:

Knowledge and understanding:

On completion of this course the student will be able to
- recognize and use the main notions of the measure theory,
- select an appropriate method for computing measure and Lebesgue integrals,
- analyze simple problems from science and technology using the measure theory.

Transferable/Key skills and other attributes:

- *Communication skills:* unambiguous and accurate expression, written and oral.
- *Calculation skills:* learning diverse calculation procedures.
- *Problem solving:* identifying and solving problems of measure theory.

• *Reševanje problemov:* prepoznavanje in reševanje problemov teorije mere.	

Metode poučevanja in učenja:	**Learning and teaching methods:**
• Predavanja • Teoretične vaje	• Lectures • Theoretical exercises

Načini ocenjevanja: **Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):	Delež (v %) / Weight (in %)	Type (examination, oral, coursework, project):
• Pisni izpit – problemi • Pisni izpit – teorija Pisni izpit - problemi se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) Pisni izpit - teorja se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.	50% 50%	• Written exam – problems • Written exam – theory Written exam – problems can be replaced by two parital tests (mid-term testing) Written exam – theory can be replaced by two parital tests (mid-term testing) Each of the mentioned commitments must be assessed with a passing grade.

Reference nosilca / Lecturer's references:

1. ARCET, Barbara, ROMANOVSKI, Valery. On some reversible cubic systems. *Mathematics*. 2021, vol. 9, no. 12, str. 1-20. ISSN 2227-7390. DOI: 10.3390/math9121446. [COBISS.SI-ID 68094211].

2. LI, Yongjun, ROMANOVSKI, Valery. Isochronous solutions of a 3-dim symmetric quadratic system. *Applied mathematics and computation*. [Print ed.]. 15 Sept. 2021, vol. 405, 12 str. ISSN 0096-3003. DOI: 10.1016/j.amc.2021.126250. [COBISS.SI-ID 95936003].

3. ARCET, Barbara, GINÉ, Jaume, ROMANOVSKI, Valery. Linearizability of planar polynomial Hamiltonian systems. *Nonlinear analysis: real world applications*. Feb. 2022, vol. 63, 19 str. ISSN 1468-1218. DOI: 10.1016/j.nonrwa.2021.103422. [COBISS.SI-ID 110154755].

4. WANG, Qinlong, YU'E, Xiong, HUANG, Wentao, ROMANOVSKI, Valery. Isolated periodic wave trains in a generalized Burgers–Huxley equation. *Electronic journal of qualitative theory of differential equations*. 2022, vol. 2022, no. 4, 16 str. ISSN 1417-3875. http://www.math.u-szeged.hu/ejqtde/p9524.pdf, DOI: 10.14232/ejqtde.2022.1.4. [COBISS.SI-ID 110159107].

5. ARCET, Barbara, ROMANOVSKI, Valery. Integrability and linearizability of symmetric three-dimensional quadratic systems. *Discrete and continuous dynamical systems. Series S*. April 2022, 18 str. ISSN 1937-1632. DOI: 10.3934/dcdss.2022104. [COBISS.SI-ID 130109955].