



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

### UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	<b>Teorija grafov</b>
<b>Course title:</b>	<b>Graph theory</b>

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Izobraževalna matematika – enopredmetna, 2. Stopnja		2.	3.
Educational mathematics - single-major, 2 <sup>nd</sup> cycle		2.	3.

**Vrsta predmeta / Course type**

obvezni / compulsory

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

Boštjan BREŠAR

**Jeziki /**

**Languages:**

**Predavanja /**

**Lectures:**

SLOVENSKO/SLOVENE

**Vaje / Tutorial:**

SLOVENSKO/SLOVENE

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Jih ni.

**Prerequisites:**

None.

**Vsebina:**

- Osnovni pojmi in primeri: graf, stopnja, izomorfizem grafov, podgrafi, povezanost, poti in cikli, dvodelni grafi, drevesa, tetivni grafi.
- Prirejanja: prirejanja in pokritja, prirejanja v dvodelnih grafih, prirejanja v splošnih grafih, Hallov poročni izrek.

**Content (Syllabus outline):**

- Basic concepts and examples: graph, degree, graph isomorphism, subgraphs, paths and cycles, trees, bipartite graphs, chordal graphs.
- Matchings: matchings and covers, matchings in bipartite graphs, matchings in general graphs, Hall's marriage theorem,
- Planar graphs: graph drawings, maps, graph dual, Euler's formula, Kuratowski theorem;

- Ravninski grafi: risbe grafov, zemljevidi, dualni graf, Eulerjeva formula, izrek Kuratowskega
- Barvanja grafov: barvanja vozlišč, Brooksov izrek, barvanja povezav, barvanja zemljevidov, izrek 4 barv.
- Eulerjevi in Hamiltonovi grafi: problem Konigsbergških mostov in Eulerjev izrek, Fleuryjev postopek, Hamiltonovi cikli in poti, potrebni in zadostni pogoji za hamiltonskost, usmerjeni grafi in turnirji, problem trgovskega potnika, problem kitajskega poštarja.
- Dodatna poglavja iz barvanja grafov: dokaz Brooksovega izreka, kritični grafi, sodobni koncepti barvanj.
- Druge grafovske invariante: povezanost in k-povezani grafi (dokaz Mengerjevega izreka); omrežja in pretoki v omrežjih; neodvisne in dominirajoče množice.
- Osnove Ramseyeve teorije: število monokromatičnih trikotnikov; Ramseyev izrek; grafovska Ramseyeva števila.

Del snovi bo prilagojen interesom in pobudam študentov ali sproti se porajajočim trendom v teoriji grafov in razvedrilni diskretni matematiki.

- Colourings of graphs: vertex colourings, Brooks' theorem, edge colourings, map colourings, 4 colour theorem.
- Eulerian and hamiltonian graphs: bridges of Konigsberg problem and Euler's theorem, Fleury's procedure, Hamilton cycles and paths, necessary and sufficient conditions for hamiltonicity, digraphs and tournaments, traveling salesman problem, Chinese postman problem.
- Additional graph colouring topics : proof of Brooks' theorem, critical graphs, modern colouring concepts.
- Other graph invariants: connectivity and k-connected graphs (proof of Menger's theorem); networks and flows in networks; independent and dominating sets.
- Basic Ramsey theory: number of monochromatic triangles; Ramsey theorem; graph Ramsey numbers.

A part of the contents will be adjusted to interests and initiative of students or to newly appearing trends in graph theory and recreational discrete mathematics.

### Temeljni literatura in viri / Readings:

- D.B. West: *Introduction to Graph Theory*, Prentice Hall, New Jersey, 2001.
- R. J. Wilson, J. J. Watkins: *Uvod v teorijo grafov*, DMFA, Ljubljana, 1997.
- R. J. Wilson: *Introduction to graph theory*, Longman, New York, 1987.
- J.A. Bondy and U.S.R. Murty: *Graph Theory*, Springer, London, 2008.

### Cilji in kompetence:

Cilj predmeta je seznaniti študente z najpomembnejšimi koncepti teorije grafov in njene uporabe ter poglobljati njihove sposobnosti za raziskovalno delo. V okviru seminarja se študent samostojno nauči izbrano snov in pripravi seminarsko predstavitev. Poseben poudarek je namenjen razvoju refleksije pridobljenega znanja in njegovega prenosa v didaktično prakso.

### Objectives and competences:

The objective of this course is to acquaint students with the most important concepts in graph theory and its application, as well as to deepen their research capabilities. For the seminar a student self-reliantly learns a chosen topic and prepares a presentation. A special emphasis is given to the reflection of acquired knowledge and its transfer to didactical practice.

**Predvideni študijski rezultati:**

Znanje in razumevanje:  
Po zaključku tega predmeta bo študent sposoben izkazati poglobljeno razumevanje osnov teorije grafov, reševati probleme, ki se v teoriji grafov pojavljajo ter pridobljeno znanje uporabljati.

Prenesljive/ključne spretnosti in drugi atributi:

- *Spretnosti komuniciranja*: ustno izražanje in javni nastop pri seminarju, ustno in pisno izražanje na izpitih
- *Reševanje problemov*: reševanje kombinatoričnih in ekstremalnih problemov v teoriji grafov.

**Intended learning outcomes:**

Knowledge and Understanding:  
On completion of this course the student will be able to demonstrate deepened understanding of graph theory basics, solve problems that appear in graph theory and apply the obtained knowledge.

Transferable/Key Skills and other attributes:

- *Communication skills*: public performance at seminar presentation, manner of expression at exams.
- *Problem solving*: solving combinatorial and extremal problems in graph theory.

**Metode poučevanja in učenja:**

- Predavanja
- Seminar
- Individualno delo

**Learning and teaching methods:**

- Lectures
- Seminar
- Individual work

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

Seminar  
Pisni izpit (naloge)  
Izpit (teorija)

Delež (v %) /  
Weight (in %)

30%  
30%  
40%

Type (examination, oral, coursework,  
project):

Seminar  
Written Exam (exercises)  
Exam (theory)

Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.

Pisni izpit – naloge se lahko nadomesti sprotimi obveznostmi (delni testi).

Pozitivna ocena pri seminarju in pisnem izpitu sta pogoja za pristop k izpitu iz teorije

Each of the mentioned commitments must be assessed with a passing grade.

Written exam – problems can be replaced by mid-term tests.

Passing grade of the seminar and of the written are required for taking the exam.

**Reference nosilca / Lecturer's references:**

1. BREŠAR, Boštjan, GASTINEAU, Nicolas, GOLOGRANC, Tanja. On a vertex-edge marking game on graphs. *Annals of combinatorics*, ISSN 0218-0006, Mar. 2021, vol. 25, iss. 1, str. 179-

194. <https://link.springer.com/article/10.1007/s00026-021-00524-9>, doi: [10.1007/s00026-021-00524-9](https://doi.org/10.1007/s00026-021-00524-9). [COBISS.SI-ID [56924419](#)].
2. BREŠAR, Boštjan, JAKOVAC, Marko, ŠTESL, Daša. Indicated coloring game on Cartesian products of graphs. *Discrete applied mathematics*, ISSN 0166-218X. [Print ed.], Jan. 2021, vol. 289, str. 320-326. <https://www.sciencedirect.com/science/article/pii/S0166218X2030500X>, doi: [10.1016/j.dam.2020.11.007](https://doi.org/10.1016/j.dam.2020.11.007). [COBISS.SI-ID [41803267](#)].
3. BREŠAR, Boštjan, BUJTÁS, Csilla, GOLOGRANC, Tanja, KLAVŽAR, Sandi, KOŠMRLJ, Gašper, MARC, Tilen, PATKÓS, Balázs, TUZA, Zsolt, VIZER, Máté. On Grundy total domination number in product graphs. *Discussiones mathematicae, Graph theory*, ISSN 1234-3099, 2021, vol. 41, no. 1, str. 225-247, ilustr. [https://www.dmgmt.uz.zgora.pl/publish/view\\_pdf.php?ID=42026](https://www.dmgmt.uz.zgora.pl/publish/view_pdf.php?ID=42026), doi: [10.7151/dmgt.2184](https://doi.org/10.7151/dmgt.2184). [COBISS.SI-ID [36071939](#)].
4. BREŠAR, Boštjan, HARTNELL, Bert L., HENNING, Michael A., KUENZEL, Kirsti, RALL, Douglas F. A new framework to approach Vizing's conjecture. *Discussiones mathematicae, Graph theory*, ISSN 1234-3099, 2021, vol. 41, no. 3, str. 749-762, ilustr. [https://www.dmgmt.uz.zgora.pl/publish/bbl\\_view\\_pdf.php?ID=48407](https://www.dmgmt.uz.zgora.pl/publish/bbl_view_pdf.php?ID=48407), doi: [10.7151/dmgt.2293](https://doi.org/10.7151/dmgt.2293). [COBISS.SI-ID [52166915](#)].
5. BREŠAR, Boštjan, GASTINEAU, Nicolas, TOGNI, Olivier. Packing colorings of subcubic outerplanar graphs. *Aequationes mathematicae*, ISSN 0001-9054, Oct. 2020, vol. 94, iss. 5, 945-967. <https://doi.org/10.1007/s00010-020-00721-6>, doi: [10.1007/s00010-020-00721-6](https://doi.org/10.1007/s00010-020-00721-6). [COBISS.SI-ID [27467011](#)].