Univerza v Mariboru

Fakulteta za naravoslovje in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	**Osnove kombinatorične optimizacije**
Course title:	**Fundamentals of Combinatorial Optimization**

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Enovit magistrski študijski program druge stopnje Predmetni učitelj		5	9
Five-year master's degree program Subject Teacher			

Vrsta predmeta / Course type	izbirni/elective
Univerzitetna koda predmeta / University course code:	

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15	30	-	-	105	6

Nosilec predmeta / Lecturer:	Drago Bokal

Jeziki / Languages:	**Predavanja / Lectures:**	Slovensko/Slovene
	Vaje / Tutorial:	Slovensko/Slovene

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	**Prerequisits:**
Vsaka izmed naštetih obveznosti v načinih ocenjevanja mora biti opravljena s pozitivno oceno. Pozitivna ocena izdelka je pogoj za pristop k ustnemu izpitu.	Each of the mentioned commitments must be assessed with a passing grade. Passing grade of the output is required for taking the oral exam.

Vsebina:	**Content (Syllabus outline):**

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja kombinatorične optimizacije: ● Diskretno odločitveno modeliranje in povezani optimizacijski problemi. ● Večkriterijska optimizacija v povezavi s konceptom najboljših razpoložljivih tehnologij.Problem nahrbtnika in njegove različice. ● Pretoki v omrežjih. Ford-Fulkersonov algoritem. ● Problem maksimalnega prirejanja. ● Problem maksimalnega prereza. ● Transportni problem. Problem kitajskega poštarja. ● Problem trgovskega potnika. ● Optimizacijski problemi v kombinatoričnih igrah. ● Spodbujevano učenje v diskretnih kontekstih. V okviru vsebine študentje izberejo zahtevnejši problem - projekt, s katerimi se poglobljeno ukvarjajo pri seminarski nalogi. Problem je povezan z njihovo bodočo kariero, zeleno in digitalno transformacijo (praktični problemi iz gospodarstva, teoretični problemi iz teorije kombinatorične optimizacije, minimiranje okoljske obremenitve, optimizacija porabe virov pri doseganju ciljnih vrednosti kazalnikov ESRS razkritij, pametnih tovarn, pametnih mest in vasi, doseganja ciljev Industrije 5.0 matematičnega programiranja in pripadajočih numeričnih algoritmov, uporaba podatkovne analitike za zagotavljanje kakovostnih podatkov za reševanje problemov kombinatorične optimizacije).Preostala predavanja se prilagodijo problemom, ki so jih izbrali študentje, in obsegajo vsebine z naslednjega seznama: ● Problem delovnega razporeda. ● Problem urnika. ● Problem razporejanja nalog enega in več strojev. ● Problemi rezanja in pakiranja. ● Diskretni modeli umetne inteligence kot kombinatorični optimizacijski problemi.	Mandatory content that familiarizes the students with fundamentals of combinatorial optimization: ● Discrete decision modelling and related optimization problems. ● Multicriterion optimization in relation to the concept of best available technologies.Knapsack problems and its variants. ● Network flows. Ford-Fulkerson's algorithm. ● Maximum matching problem. ● Maximum cut problem. ● Transport problem. Chinese postman problem. ● Travelling salesman problem. ● Optimization problems in combinatorial games. ● Reinforcement learning in discrete contexts. Within the coursework, the students select a problem - project whose result is coursework report. The problem is related to their future career, green and digital transformation (practical problems from industry and business, theoretical problems from the theory of combinatorial optimization, minimizing environmental footprint, optimizing resource usage achieving goal values of indices of ESRS disclosures, smart factories, smart cities and villages, reaching goals of Industry 5.0, the areas of optimization, algorithms, modelling, using data analytics for monitoring data quality solving problems of combinatorial optimization).The content of the remaining lectures is selected according to these projects from the following list: ● Employee timetabling problem. ● School timetabling problems. ● Scheduling tasks of one or several processors. ● Cutting and packing problems. ● Discrete artificial intelligence models as combinatorial optimization problems. ● Using concepts and methods of combinatorial optimization for solving problems of green transformation, smart factories, smart cities.

● Uporaba konceptov in metod kombinatorične optimizacije za reševanje problemov zelene transformacije, pametnih tovarn, pametnih mest. ● Uporaba konceptov in metod kombinatorične optimizacije v povezavi s tehnikami in izzivi umetne inteligence in digitalne transformacije. ● Drugi s kombinatorično optimizacijo povezani problemi, usklajeni s študentskimi interesi.	● Using concepts and methods of combinatorial optimization related to techniques and challenges of artificial intelligence and digital transformation. ● Other problems, related to combinatirial optimization and student's interest.

Temeljni literatura in viri / Readings:

1. Du, D. Z., Pardalos, P. M., Hu, X., & Wu, W. (2022). *Introduction to combinatorial optimization* (Vol. 196). Springer Nature.
2. B. Robič: Aproksimacijski algoritmi, Založba FRI, Ljubljana 2002.
3. R. Wilson, M. Watkins, Uvod v teorijo grafov, DMFA, Ljubljana 1997.

Dodatna literatura / Additional Readings:

1. J.Žerovnik: Osnove teorije grafov in diskretne optimizacije (druga izdaja), Fakulteta za strojništvo, Maribor 2005.
2. E. Zakrajšek: Matematično modeliranje, DMFA, Ljubljana 2004.
3. B. Korte, J. Vygen: Combinatorial Optimization, Theory and Algorithms, Springer, Berlin 2000.
4. D. Cvetković, V. Kovačević-Vujčić: Kombinatorna optimizacija, DOPIS Beograd 1996.
5. S. Zlobec, J. Petrić: Nelinearno programiranje, Naučna knjiga, Beograd 1989.
6. E. Kreyszig: Advanced Engineering Mathematics, (seventh edition), Wiley, New York 1993.

Cilji in kompetence:	Objectives and competences:
Uporabiti proces matematičnega modeliranja na diskretnih optimizacijskih problemih. Vrednotiti kompetenco samostojnega apliciranja uporabo matematičnih metod na probleme iz finančne optimizacije, ekonomije, ter širše iz gospodarstva. Izbrati in integrirati tehnološka orodja, s katerimi se srečujemo pri reševanju optimizacijskih problemov in problemov matematičnega modeliranja. Spoznati Okvir digitalnih kompetenc za državljane in Evropski kompetenčni okvir za trajnostnost kot primer učnih prostorov.	Apply the process of mathematical modelling of discrete optimization problems. Evaluate skills of independent application of mathematical methods to the problems from financial optimization, economics, and broader from industry. Choose and integrate with technological tools that assist solving optimization problems and problems related to mathematical modelling. Learn about The Digital Competence Framework For Citizens and GreenComp competence framework as examples of learning spaces. Recognize most relevant course related

Prepoznati bistvene s predmetom povezane kompetence v navedenih kompetenčnih okvirih in samooceniti svoje znanje. Zaznati potrebo po nadgradnji izbranih kompetenc omenjenih okvirov in pridobiti informacije za njihovo nadgrajevanje.	competences in the above competence frameworks and self-evaluate their knowledge of these competences. Recognize the need for upgrading the selected competences of these frameworks and obtain relevant information for their upgrading.

Predvideni študijski rezultati:	**Intended learning outcomes:**
Znanje in razumevanje: ● Uporaba principov kombinatorične optimizacije. ● Izbor in analiza sodobnih metod za reševanje problemov kombinatorične optimizacije. ● Prilagajanje diskretnih modelov in drugih aplikacij kombinatorične optimizacije v finančni matematiki, optimiranju virov, in širše. Prenesljive/ključne spretnosti in drugi atributi: ● Uporaba metod kombinatorične optimizacije v finančni matematiki, ekonomiji, poslovnih vedah, inžinirstvu, fiziki in številnih drugih družboslovnih in naravoslovnih vedah. ● Prilagajanje tehnik kombinatorične optimizacije v problemih s področja finančne optimizacije, ekonomije in širše. ● Izdelava in vrednotenje temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.	Knowledge and understanding: ● Applying principles of combinatorial optimization. ● Selection and analysis of modern methods for solving combinatorial optimization problems. ● Adaptation of discrete models and other advanced applications of combinatorial optimization in financial optimization, resource optimization, and wider. Transferable/Key Skills and other attributes: ● Applications of methods of combinatorial optimization in finacial mathematics, economy, business, engineering, physics, and numerous other social and natural sciences. ● Adaptation of the techniques of combinatorial optimization in problems from financial optimization, economics, and wider. ● Preparation and evaluation of a detailed technical report or focused report paper describing a mathematical model of a specific mathematical problem the students encounter while investigation their possible future careers. ●

Metode poučevanja in učenja:	**Learning and teaching methods:**
● Na predavanjih študentje spoznajo predpisano snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti.	● At the lectures, the students are familiarized with the required contents of the course. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course.

● Študentski projekti so osrednji element predmeta. Študentje se jih lotevajo v skladu s smernicami projektnega in sodelovalnega učenja. ● V okviru seminarskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja in sodelovalnega učenja delajo na izbranih projektih. ● V okviru seminarja študentje uporabljajo, analizirajo in vrednotijo svoje projekte za potrebe ustvarjanja novih rešitev za okolje, iz katerega izvirajo problemi. ● Z uporabo tehnik izkušanja matematičnega modeliranja študenti merijo svoj napredek pri predmetu, ga reflektirajo in tako uporabljajo njegov matematični model za spodbujanje sprotnega dela, opazovanje izboljšanja, ki izvira iz matematično podprtega odločanja. ● Dve predstavitvi njihovih rezultatov pomagajo študentom priučiti se suverenosti javnega nastopanja in zagovarjanja doseženih rezultatov. Osredotočeni dvodnevni organizirani hackathon pomaga študentom izkusiti veselje ob osredotočenem raziskovanju in sodelovanju z vrstniki. ● obrnjeno učenje se izkaže s pomočjo formativnega spremljanja študentskih skupin, ki predstavijo napredek zadnjega tedna, mentor pa jih usmeri k aktivnostim višjih taksonov Bloomove taksonomije. ●	● Student projects are the central element of the course. Students work on them according the principles of project-based learning and collaborative learning. ● Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning and collaboration learning on projects of their choice. ● At the seminar, the students apply, analyse, and evaluate their projects in order to create new solutions desired by the environment the problems are coming from. ● Applying embedded mathematical modelling, the students measure and reflect upon their progress thus encouraging ongoing work, using its mathematical model to experience improvement coming from mathematically supported decision making. ● Three presentations of their results help students acquire confidence with public presentation and defending their results. ● Focused two-day workshop-style organized hackathon helps students experience the joy of focused research and collaboration with peers. ● Flipped learning is expressed through formative assessment of student groups, which discuss progress of the previous week and the mentor directs them towards activities higher in the Bloom taxonomy. ●
Načini ocenjevanja:	**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):	Delež (v %) / Weight (in %)	Type (examination, oral, coursework, project):
Izdelek	75%,	Output
Ustni izpit	25%	Oral exam

Reference nosilca / Lecturer's references:

BOKAL, Drago, DVOŘÁK, Zdeněk, HLINĚNÝ, Petr, LEANÕS, Jesús, MOHAR, Bojan, WIEDERA, Tilo. Bounded degree conjecture holds precisely for c-crossing-critical graphs with c≤12
. *Combinatorica*. Oct. 2022, vol. 42, iss. 5, str. 701-728. ISSN 0209-9683. https://link.springer.com/article/10.1007/s00493-021-4285-3, DOI: 10.1007/s00493-021-4285-3. [COBISS.SI-ID 213684739]

BOKAL, Drago, CHIMANI, Markus, NOVER, Alexander, SCHIERBAUM, Jöran, STOLZMANN, Tobias, WAGNER, Mirko H., WIEDERA, Tilo. Properties of large 2-crossing-critical graphs. *Journal of graph algorithms and applications*. 2022, vol. 26, no. 1, str. 111-147. ISSN 1526-1719. Digitalna knjižnica Univerze v Mariboru – DKUM, DOI: 10.7155/jgaa.00585, DOI: 20.500.12556/DKUM-88780. [COBISS.SI-ID 144719363]

SMOLE, Andreja, JAGRIČ, Timotej, BOKAL, Drago. Principal/Two-Agent model with internal signal. *Central European journal of operations research*. sep. 2021, vol. 29, no. 3, str. 791-808, ilustr. ISSN 1435-246X. https://link.springer.com/content/pdf/10.1007/s10100-020-00719-0.pdf, DOI: 10.1007/s10100-020-00719-0. [COBISS.SI-ID 75068163]

BOKAL, Drago, STEINBACHER, Mitja. Phases of psychologically optimal learning experience : task-based time allocation model. *Central European Journal of Operations Research*, ISSN 1435-246X, 2019, str. 1-23, doi: 10.1007/s10100-019-00609-0. [COBISS.SI-ID 24408328]

BOKAL, Drago, BREŠAR, Boštjan, JEREBIC, Janja. A generalization of Hungarian method and Hall's theorem with applications in wireless sensor networks. Discrete Applied Mathematics, ISSN 0166-218X. [Print ed.], 2012, vol. 160, iss. 4-5, str. 460-470. http://dx.doi.org/10.1016/j.dam.2011.11.007. [COBISS.SI-ID 16191577]

ČAMPULOVÁ, Martina, MICHÁLEK, Jaroslav, MIKUŠKA, Pavel, BOKAL, Drago. Nonparametric algorithm for identification of outliers in environmental data. *Journal of chemometrics*, ISSN 0886-9383, 2018, str. 1-17, graf. prikazi, doi: 10.1002/cem.2997. [COBISS.SI-ID 23647240]

BAŠIĆ, Nino, BOKAL, Drago, BOOTHBY, Tomas, RUS, Jernej. An algebraic approach to enumerating non-equivalent double traces in graphs. *MATCH Communications in Mathematical and in Computer Chemistry*, ISSN 0340-6253, 2017, vol. 78, no. 3, str. 581-594. http://match.pmf.kg.ac.rs/electronic_versions/Match78/n3/match78n3_581-594.pdf. [COBISS.SI-ID 18108505]