



Univerza v Mariboru

Fakulteta za naravoslovje
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS

Predmet: Operacijske raziskave
Course title: Operations Research

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Enovit magistrski študijski program druge stopnje Predmetni učitelj	/	5.	9.
Five-year master's degree program Subject Teacher	/		

Vrsta predmeta / Course type

izbirni/elective

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15	30	-	-	105	6

Nosilec predmeta / Lecturer:

Drago Bokal

Jeziki /

Languages:

Predavanja /

Lectures:

Vaje / Tutorial:

Slovensko/Slovene

Slovensko/Slovene

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Poznavanje enostavnih algoritmov.
Poznavanje osnov linearne algebre in vektorske analize.
Vsaka izmed naštetih obveznosti v načinih ocenjevanja mora biti opravljena s pozitivno oceno.
Pozitivna ocena izdelka je pogoj za pristop k ustnemu izpitu.

Prerequisites:

Knowledge of simple algorithms.
Knowledge of basic linear algebra and calculus.

Each of the mentioned commitments must be assessed with a passing grade.
Passing grade of the output is required for taking the oral exam.

Vsebina:

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav in matematičnih programov:

- Stohastični Markovski odločitveni procesi. Markovske verige. Markovsko jedro. Uporaba v univerzalnem modelu procesa.
- Uporaba Markovskih jeder v umetni inteligenci.
- Nelinearno, kvadratično programiranje. Lagrangeovske metode in metoda aktivne množice. Programi z linearnimi vezmi. Cikcakanje. LagrangeNewtonova metoda.
- Semidefinitno programiranje. Aplikacije v kombinatorični optimizaciji.
 - Uporaba konceptov in metod operacijskih raziskav za reševanje problemov zelene transformacije, pametnih tovarn, pametnih mest.
 - Uporaba konceptov in metod operacijskih raziskav v povezavi s tehnikami in izzivi umetne inteligence in digitalne transformacije.

V okviru vsebine študentje izberejo zahtevnejši problem - projekt, s katerimi se poglobljeno ukvarjajo pri seminarski nalogi. Problem je povezan z njihovo bodočo kariero, zeleno in digitalno transformacijo (praktični problemi iz gospodarstva, teoretični problemi iz teorije operacijskih raziskav, minimiranje okoljske obremenitve, optimizacija porabe virov pri doseganju ciljnih vrednosti kazalnikov ESRS razkritij, pametnih tovarn, pametnih mest in vasi, doseganja ciljev Industrije 5.0, uporaba podatkovne analitike za zagotavljanje kakovostnih podatkov za reševanje problemov operacijskih raziskav). Preostala predavanja se prilagodijo problemom, ki so jih izbrali študentje, in obsegajo vsebine z naslednjega seznama:

- Nevezani ekstrem, Newtonova metoda.
- Vezani ekstrem. Lagrangeovi multiplikatorji. Potrebni in zadostni pogoji za nastop vezanega lokalnega

Content (Syllabus outline):

Mandatory content that familiarizes the students with fundamentals of operations research and mathematical programs:

- Stochastic Markov decision processes. Markov chains. Markov Kernels. Applications in universal process model.
- Applications of Markov kernels in artificial intelligence.
- Nonlinear, quadratic programming. Lagrange methods and active set methods. Programs with linear constraints. Zigzagging. Lagrange-Newton method.
- Semidefinite programming. Applications in combinatorial optimization.
- Using concepts and methods of operations research for solving problems of green transformation, smart factories, smart cities.
- Using concepts and methods of operations research related to techniques and challenges of artificial intelligence and digital transformation.

Within the coursework, the students select a problem - project whose result is coursework report. The problem is related to their future career, green and digital transition (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling, minimizing environmental impact, optimizing resource usage achieving goal values of ESRS disclosures, smart factories, smart cities and villages, achieving goals of Industry 5.0, applying data analytics to ascertain quality data for solving problems with operations research). The content of the remaining lectures is selected according to these projects from the following list:

- Unconstrained optimization. Newton's method.
- Constrained optimization. Lagrange multipliers. Necessary and sufficient conditions for a constrained local optimum. Dual of a convex program.
- Price of robustness robust optimization method.

ekstrema. Wolfe-ov dual konveksnega programa. ● Robustna optimizacija po metodi cene robustnosti. ● Imunizacija portfelja in stohastično programiranje. ● Stohastično nelinearno programiranje (diskretna in zvezna slučajna spremenljivka). Dekompozicija. ● Stožčasto programiranje. Lorentzov in semidefinitni stožec. Stožčasto kvadratično programiranje. ● Aplikacije semidefinitnega programiranja: kvadratični problem prirejanja, problem trgovskega potnika, problem maksimalnega prereza grafa. ● Aplikaciji stohastičnega programiranja: Markowitzevi modeli optimizacije portfelja, modeli večfaznega stohastičnega načrtovanja. ● Modeli največjega verjetja, metoda najmanjših kvadratov, umerjanje modelov na znane podatke, inverzni problemi, druge podatkovne analize. ● Optimizacijski matematični modeli s področja kontrolnih sistemov, obdelave signalov. ● Metoda podpornih vektorjev. Druge vsebine s področja operacijskih raziskav in matematičnega modeliranja, povezane s študentskimi projekti. V okviru projektnih nalog se študentje srečajo tudi s programsko opremo za matematično modeliranje. komercialno (Excel, Lindo, Matlab) oz. prostodostopno in odprtokodno (Python, SciLab, NEOS, R).	● Portfolio immunization using stochastic programming. ● Stochastic nonlinear programming (discrete and continuous stochastic variables). Decomposition. ● Conic programming. Lorentz and semidefinite cone. Conic quadratic programming. ● Applications of semidefinite programming: quadratic assignment problem, travelling salesman problem, max cut problem. ● Applications of stochastic programming: Markowitz models of portfolio optimization, multiperiod stochastic planning models. ● Maximum likelihood models, least squares method, parameter fitting for given data. ● Optimization mathematical models from control theory and signal processing. ● Support Vector Machine. ● Other content from the domain of operations research and mathematical programming, related to students' problems. Within their coursework and exercises, the students familiarize themselves with software for mathematical modelling, either commercial (Excel, Lindo, Matlab) or freely available open source (Python, SciLab, Neos, R).
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Temeljni literatura in viri / Readings:

1. J. Franklin, Methods of Mathematical Economics: Linear and Nonlinear Programming, Fixed-Point Theorems. Classics in Applied Mathematics 37, SIAM, 2002.
2. R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.
3. Novak, T., Povh, J., & Žerovnik, J. (2020). Izbrana poglavja iz operacijskih raziskav (str. 227). Fakulteta za strojništvo.

Dodatna literatura / Additional Readings:

1. J. Curwin, R. Slater. Quantitative Methods for Business Decisions. Third Edition. Chapman & Hall, London, 1991.

2. L. Neralić, Uvod u matematičko programiranje 1. Udžbenici Sveučilišta u Zagrebu, Zagreb, 2001.
3. S. A. Zenios, Financial Optimization. Cambridge University Press, Cambridge, 1993.

Cilji in kompetence:

Izvajati proces matematičnega modeliranja na zveznih optimizacijskih problemih.

Uporabiti matematične metode na problemih iz finančne optimizacije, ekonomije, ter širše iz gospodarstva.

Uporabiti tehnološka orodja, s katerimi se srečujemo pri reševanju optimizacijskih problemov in problemov matematičnega modeliranja.

Spoznati Okvir digitalnih kompetenc za državljane 2.2 in Evropski kompetenčni okvir za trajnostnost kot primer učnih prostorov.

Prepoznati bistvene kompetence v navedenih kompetenčnih okvirih in samooceniti svoje znanje.

Zaznati potrebo po nadgradnji izbranih kompetenc omenjenih okvirov in pridobiti informacije za njihovo nadgrajevanje.

Objectives and competences:

Execute the process of mathematical modelling of continuous optimization problems.

Apply mathematical methods to the problems from financial optimization, economics, and broader from industry.

Use technological tools that assist solving optimization problems and problems related to mathematical modelling.

Learn about The Digital Competence Framework For Citizens and GreenComp competence framework as examples of learning spaces.

Recognize most relevant competences in the above competence frameworks and self-evaluate their knowledge of these competences.

Recognize the need for upgrading the selected competences of these frameworks and obtain relevant information for their upgrading.

Predvideni študijski rezultati:

Znanje in razumevanje:

- Diskutirati o zahtevnejših principih matematičnega programiranja.
- Uporabiti sodobne numerične metode za reševanje matematičnih programov.
- Ilustrirati Markowitzeve modele in druge zahtevne aplikacije matematičnega programiranja v finančni optimizaciji in širše.

Prenesljive/ključne spretnosti in drugi atributi:

Intended learning outcomes:

Knowledge and Understanding:

- Discussing advanced principles of mathematical programming.
- Applying modern numerical methods for solving mathematical programs.
- Illustrating Markowitz models and other advanced applications of mathematical programming, financial optimization and wider.

Transferable/Key Skills and other attributes:

• Uporaba modelov in tehnik operacijskih raziskav v finančni matematiki, ekonomiji, poslovnih vedah, inženirstvu, fiziki in številnih drugih družboslovnih in naravoslovnih vedah. • Uporaba tehnik matematičnega programiranja v problemih s področja finančne optimizacije, ekonomije in širše. • Izdelava temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.	• Applications of models and techniques of Operations Research in financial mathematics, economy, business, engineering, physics, and numerous other social and natural sciences. • Applications of its techniques in problems from financial optimization, economics, and wider. • Preparing detailed technical report of applying a mathematical model on specific problem, related to students future career.
Metode poučevanja in učenja:	Learning and teaching methods:
• Na predavanjih študentje spoznajo predpisano snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti. • Študentski projekti so osrednji element predmeta. Študentje se jih lotevajo v skladu s smernicami projektnega in sodelovalnega učenja. • V okviru seminarskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja in sodelovalnega učenja delajo na izbranih projektih. • V okviru seminarja študentje uporabljajo, analizirajo in vrednotijo svoje projekte za potrebe ustvarjanja novih rešitev za okolje, iz katerega izvirajo problemi. • Z uporabo tehnik izkušanja matematičnega modeliranja študenti merijo svoj napredek pri predmetu, ga reflektirajo in tako uporabljajo njegov matematični model za spodbujanje sprotne dela, opazovanje izboljšanja, ki izvira iz matematično podprtega odločanja.	At the lectures, the students are familiarized with the required contents of the course. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course. • Student projects are the central element of the course. Students work on them according the principles of project-based learning and collaborative learning. • Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning and collaboration learning on projects of their choice. • At the seminar, the students apply, analyse, and evaluate their projects in order to create new solutions desired by the environment the problems are coming from. • Applying embedded mathematical modelling, the students measure and reflect upon their progress thus encouraging ongoing work, using its mathematical model to experience

• Dve predstavitvi njihovih rezultatov pomagajo študentom priučiti se suverenosti javnega nastopanja in zagovarjanja doseženih rezultatov. Osredotočeni dvodnevni organizirani hackathon pomaga študentom izkusiti veselje ob osredotočenem raziskovanju in sodelovanju z vrstniki. • Obrnjeno učenje se izkaže s pomočjo formativnega spremljanja študentskih skupin, ki predstavijo napredek zadnjega tedna, mentor pa jih usmeri k aktivnostim višjih taksonov Bloomove taksonomije.	improvement coming from mathematically supported decision making. • Three presentations of their results help students acquire confidence with public presentation and defending their results. • Focused two-day workshop-style organized hackathon helps students experience the joy of focused research and collaboration with peers. • Flipped learning is expressed through formative assessment of student groups, which discuss progress of the previous week and the mentor directs them towards activities higher in the Bloom taxonomy.		
Načini ocenjevanja:	Assessment:		
Način (pisni izpit, ustno izpraševanje, naloge, projekt): Izdelek Ustni izpit	<table border="1"> <tr> <td data-bbox="730 956 935 1169"> Delež (v %) / Weight (in %) 75% 25% </td><td data-bbox="935 956 1476 1169"> Type (examination, oral, coursework, project): Output Oral exam </td></tr> </table>	Delež (v %) / Weight (in %) 75% 25%	Type (examination, oral, coursework, project): Output Oral exam
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Reference nosilca / Lecturer's references:

BOKAL, Drago, DVOŘÁK, Zdeněk, HLINĚNÝ, Petr, LEANŐS, Jesús, MOHAR, Bojan, WIEDERA, Tilo. Bounded degree conjecture holds precisely for c-crossing-critical graphs with $c \leq 12$. *Combinatorica*. Oct. 2022, vol. 42, iss. 5, str. 701-728. ISSN 0209-9683.

BOKAL, Drago, CHIMANI, Markus, NOVER, Alexander, SCHIERBAUM, Jöran, STOLZMANN, Tobias, WAGNER, Mirko H., WIEDERA, Tilo. Properties of large 2-crossing-critical graphs. *Journal of graph algorithms and applications*. 2022, vol. 26, no. 1, str. 111-147. ISSN 1526-1719. DOI: 10.7155/jgaa.00585. [COBISS.SI-ID 144719363]

BOKAL, Drago, JEREbic, Janja. Guarding a subgraph as a tool in pursuit-evasion games. *Discussiones mathematicae. Graph theory*. 2022, vol. 42, no. 1, str. 123-138. ISSN 12343099. https://www.dmg.t.uz.zgora.pl/publish/view_vol_article.php?E8F29296C3FB4B933A2E. [COBISS.SI-ID 8147219]

SMOLE, Andreja, JAGRIČ, Timotej, BOKAL, Drago. Principal/Two-Agent model with internal signal. *Central European journal of operations research*. sep. 2021, vol. 29, no. 3, str. 791-808, ilustr. ISSN 1435-246X. <https://link.springer.com/content/pdf/10.1007/s10100-020-00719-0.pdf>, DOI: 10.1007/s10100-020-00719-0. [COBISS.SI-ID 75068163]

VEGI KALAMAR, Alen, ŽERAK, Tadej, BOKAL, Drago. Counting Hamiltonian cycles in 2-tiled graphs. *Mathematics*. 2021, vol. 9, iss. 6, str. 1-27. ISSN 2227-7390. DOI: [10.3390/math9060693](https://doi.org/10.3390/math9060693). [COBISS.SI-ID [61574403](https://www.cobiss.si/id/61574403)]

