



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

### UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	<b>Operacijske raziskave</b>
<b>Course title:</b>	<b>Operations Research</b>

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Enovit magistrski študijski program druge stopnje Predmetni učitelj	/	5.	9.
Five-year master's degree program Subject Teacher	/		

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15	30	-	-	105	6

**Nosilec predmeta / Lecturer:**

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	Slovensko/Slovene
	<b>Vaje / Tutorial:</b>	Slovensko/Slovene

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Poznavanje enostavnih algoritmov.  
Poznavanje osnov linearne algebre in vektorske analize.

**Prerequisites:**

Knowledge of simple algorithms.  
Knowledge of basic linear algebra and calculus.

**Vsebina:**

**Content (Syllabus outline):**

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav in matematičnih programov:

- Nevezani ekstrem, Newtonova metoda.
- Vezani ekstrem. Lagrangeovi multiplikatoriji. Potrebni in zadostni pogoji za nastop vezanega lokalnega ekstrema. Wolfe-ov dual konveksnega programa.
- Metoda podpornih vektorjev.
- Metoda notranje točke za linearno programiranje. Dokaz obstoja centralne poti. Primarno-dualna metoda sledenja centralni poti.
- Aplikacije nadaljevalnih optimizacijskih problemov pri umetni inteligenci. Odvedljivo programiranje.
- Matematični temelji globokega učenja.

V okviru vsebine študentje izberejo zahtevnejši problem - projekt, s katerimi se poglobljeno ukvarjajo pri seminarski nalogi. Problem je povezan z njihovo bodočo kariero (praktični problemi iz gospodarstva, teoretični problemi iz teorije matematičnega programiranja in pripadajočih numeričnih algoritmov). Preostala predavanja se prilagodijo problemom, ki so jih izbrali študentje, in obsegajo vsebine z naslednjega seznama:

- Nelinearno programiranje. Kazenska in odbojna funkcija. Lagrange-Newtonova metoda (SQP).
- Kvadratično programiranje. Lagrangeovske metode in metoda aktivne množice. Programi z linearnimi vezmi. Cikcakanje.
- Semidefinitno programiranje. Aplikacije v kombinatorični optimizaciji.
- Stožčasto programiranje. Lorentzov in semidefinitni stožec. Stožčasto kvadratično programiranje.
- Robustna optimizacija po metodi cene robustnosti.
- Imunizacija portfelja in stohastično programiranje.

Mandatory content that familiarizes the students with fundamentals of operations research and mathematical programs:

- Unconstrained optimization. Newton's method.
- Constrained optimization. Lagrange multipliers. Necessary and sufficient conditions for a constrained local optimum. Dual of a convex program.
- Support Vector Machine.
- Interior point methods for linear programming. Existence of the central path. Primal-dual methods of following the central path.
- Applications of continuous optimization problems in artificial intelligence. Differentiable programming.
- Mathematical fundamentals of deep learning.

Within the coursework, the students select a problem - project whose result is coursework report. The problem is related to their future career (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list:

- Nonlinear programming. Penalty and barrier functions. Lagrange-Newton method. Sequential Quadratic Programming.
- Quadratic programming. Lagrange methods and active set methods. Programs with linear constraints. Zigzagging.
- Semidefinite programming. Applications in combinatorial optimization.
- Conic programming. Lorentz and semidefinite cone. Conic quadratic programming.
- Price of robustness robust optimization method.
- Portfolio immunization using stochastic programming.

<ul style="list-style-type: none"> <li>● Stohastično nelinearno programiranje (diskretna in zvezna slučajna spremenljivka). Dekompozicija.</li> <li>● Aplikacije semidefinitnega programiranja: kvadratični problem prirejanja, problem trgovskega potnika, problem maksimalnega prereza grafa.</li> <li>● Aplikaciji stohastičnega programiranja: Markowitzevi modeli optimizacije portfelja, modeli večfaznega stohastičnega načrtovanja.</li> <li>● Modeli največjega verjetja, metoda najmanjših kvadratov, umerjanje modelov na znane podatke, inverzni problemi, druge podatkovne analize.</li> <li>● Optimizacijski matematični modeli s področja kontrolnih sistemov, obdelave signalov.</li> <li>● Druge vsebine s področja operacijskih raziskav in matematičnega modeliranja, povezane s študentskimi projekti.</li> </ul> <p>V okviru seminarских nalog se študentje srečajo tudi s programsko opremo za matematično modeliranje. komercialno (Excel, Lindo, Matlab) oz. prostodostopno in odprtokodno (Python, SciLab, NEOS, R).</p>	<ul style="list-style-type: none"> <li>● Stochastic nonlinear programming (discrete and continuous stochastic variables). Decomposition.</li> <li>● Applications of semidefinite programming: quadratic assignment problem, travelling salesman problem, max cut problem.</li> <li>● Applications of stochastic programming: Markowitz models of portfolio optimization, multiperiod stochastic planning models.</li> <li>● Maximum likelihood models, least squares method, parameter fitting for given data.</li> <li>● Optimization mathematical models from control theory and signal processing.</li> <li>● Other content from the domain of operations research and mathematical programming, related to students' problems.</li> </ul> <p>Within their coursework and exercises, the students familiarize themselves with software for mathematical modelling, either commercial (Excel, Lindo, Matlab) or freely available open source (Python, SciLab, Neos, R).</p>
---	---

### Temeljni literatura in viri / Readings:

R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.

J. Curwin, R. Slater. Quantitive Methods for Business Decisions. Third Edition. Chapman & Hall, London, 1991.

S. A. Zenios, Financial Optimization. Cambridge University Press, Cambridge, 1993.

R. Fletcher, Practical Methods of Optimization. Second Edition. Wiley, Chichester, 2001.

A. Ben-Tal, A. Nemirowski: Lectures on modern convex optimization. H. Milton Stewart School of Industrial & Systems Engineering, Georgia Institute of Technology, Atlanta, 2012.

C. Huang, R. H. Litzenberger. Foundations for Finacial Economics. Prentice Hall, Inc., Upper Saddle River, New Jersey, 1988.

P. Kall, S. W. Wallace. Stochastic Programming. Wiley, Chichester, 1994.

L. Neralić, Uvod u matematičko programiranje 1. Udžbenici Sveučilišta u Zagrebu, Zagreb, 2001.

R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.

J. Renegar. A Mathematical View of Interior-Point Methods in Convex Optimization. MPS-SIAM Series on Optimization. SIAM, Philadelphia, 2001.

S. A. Zenios, Financial Optimization. Cambridge University Press, Cambridge, 1993.

**Cilji in kompetence:**

Usvojiti proces matematičnega modeliranja na zveznih optimizacijskih problemih.

Razviti kompetenco samostojnega apliciranja matematičnih metod na probleme iz finančne optimizacije, ekonomije, ter širše iz gospodarstva.

Spoznati tehnološka orodja, s katerimi se srečujemo pri reševanju optimizacijskih problemov in problemov matematičnega modeliranja.

**Objectives and competences:**

Familiarize the students with the process of mathematical modelling of continuous optimization problems.

Develop competent skills of independent application of mathematical methods to the problems from financial optimization, economics, and broader from industry.

Familiarize the students with technological tools that assist solving optimization problems and problems related to mathematical modelling.

**Predvideni študijski rezultati:**

Znanje in razumevanje:

- Razumevanje zahtevnejših principov matematičnega programiranja.
- Poglobi znanje iz sodobnih numeričnih metod za reševanje matematičnih programov.
- Poglobiti znanje iz Markowitzevih modelov in drugih zahtevnih aplikacij matematičnega programiranja v finančni optimizaciji in širše.

Prenosljive/ključne spretnosti in drugi atributi:

- Direktne aplikacije v finančni matematiki, ekonomiji, poslovnih vedah, inženirstvu, fiziki in številnih drugih družboslovnih in naravoslovnih vedah.
- Suvereno obvladovanje procesa matematičnega modeliranja in uporabe tehnik matematičnega programiranja v problemih s področja finančne optimizacije, ekonomije in širše.
- Izdelava temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.

**Intended learning outcomes:**

Knowledge and Understanding:

- To be able to understand advanced principles of mathematical programming.
- To deepen the knowledge of modern numerical methods for solving mathematical programs.
- To deepen the knowledge of details of Markowitz models and other advanced applications of mathematical programming, financial optimization and wider.

Transferable/Key Skills and other attributes:

- Direct applications in financial mathematics, economy, business, engineering, physics, and numerous other social and natural sciences.
- Competent mastering of the process of mathematical modelling and applications of its techniques in problems from financial optimization, economics, and wider.

### Metode poučevanja in učenja:

- Pri predavanjih študentje spoznajo snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti.
- V okviru seminarских vaj študentje razumevanje snovi utrjujejo na večjem projektu, povezanem z njihovo bodočo kariero. Razporejeni so v večje skupine, ki po metodah problemskega učenja obravnavajo izbrani problem od zbiranja podatkov, preko razvoja modela, izbora in prilagajanja ustreznih tehnoloških rešitev do razmisleka o implementaciji rešitve. Koncept poučevanja je podrobneje predstavljen kot ciljni aplikativni predmet.
- V okviru seminarja študentje uporabljajo, analizirajo in vrednotijo svoje projekte za potrebe ustvarjanja novih rešitev za okolje, iz katerega izvirajo problemi.
- Z uporabo izkušanja matematičnega modeliranja študenti merijo in razmišljajo o svojem napredku in tako uporabljajo svoj matematični model za opazovanje izboljšanja, ki izvira iz matematično podprtega odločanja.
- Tri predstavitve njihovih rezultatov pomagajo študentom priučiti se suverenosti javnega nastopanja in zagovarjanja svojih rezultatov.
- Osredotočeni dvodnevni organizirani hackathon pomaga študentom izkusiti veselje ob osredotočenih raziskovanjih in sodelovanju z vrstniki.

### Learning and teaching methods:

- At the lectures, the students are familiarized with the course content. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course.
- At the tutorials, the student deepen their understanding of the material by working on an extensive problem related to their future career. They are organized in larger groups who research the chosen problem guided by methodologies of problem-based learning. Within solving the problem, they experience all the stages from requirements and data gathering, model development, selecting and adapting technological solutions to discussing various aspects of implementation of the results.
- At the seminar, the students apply, analyse, and evaluate their projects in order to create new solutions desired by the environment the problems are coming from.
- Applying embedded mathematical modelling, the students measure and reflect upon their progress thus using its mathematical model to experience improvement coming from mathematically supported decision making.
- Three presentations of their results help students acquire confidence with public presentation and defending their results.
- Focused two-day workshop-style organized hackathon helps students

		experience the joy of focused research and collaboration with peers.
<b>Načini ocenjevanja:</b>		<b>Assessment:</b>
Način (pisni izpit, ustno izpraševanje, naloge, projekt): Seminarska naloga Ustni izpit	Delež (v %) / Weight (in %) 80% 20%	Type (examination, oral, coursework, project): Coursework report Oral exam
Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.		Each of the mentioned commitments must be assessed with a passing grade.
Pozitivna ocena pri seminarski nalogi je pogoj za pristop k izpitu.		Passing grade of the coursework report is required for taking the exam.

**Reference nosilca / Lecturer's references:**

SMOLE, Andreja, JAGRIČ, Timotej, BOKAL, Drago. Principal/Two-Agent model with internal signal. *Central European journal of operations research*. sep. 2021, vol. 29, no. 3, str. 791-808, ilustr. ISSN 1435-246X. <https://link.springer.com/content/pdf/10.1007/s10100-020-00719-0.pdf>, DOI: [10.1007/s10100-020-00719-0](https://doi.org/10.1007/s10100-020-00719-0). [COBISS.SI-ID [75068163](https://www.cobiss.si/id/75068163)]

BOKAL, Drago, STEINBACHER, Mitja. Phases of psychologically optimal learning experience : task-based time allocation model. *Central European Journal of Operations Research*, ISSN 1435-246X, 2019, str. 1-23, doi: [10.1007/s10100-019-00609-0](https://doi.org/10.1007/s10100-019-00609-0). [COBISS.SI-ID [24408328](https://www.cobiss.si/id/24408328)]

ČAMPULOVÁ, Martina, MICHÁLEK, Jaroslav, MIKUŠKA, Pavel, BOKAL, Drago. Nonparametric algorithm for identification of outliers in environmental data. *Journal of chemometrics*, ISSN 0886-9383, 2018, str. 1-17, graf. prikazi, doi: [10.1002/cem.2997](https://doi.org/10.1002/cem.2997). [COBISS.SI-ID [23647240](https://www.cobiss.si/id/23647240)]

BAŠIĆ, Nino, BOKAL, Drago, BOOTHBY, Tomas, RUS, Jernej. An algebraic approach to enumerating non-equivalent double traces in graphs. *MATCH Communications in Mathematical and in Computer Chemistry*, ISSN 0340-6253, 2017, vol. 78, no. 3, str. 581-594. [http://match.pmf.kg.ac.rs/electronic\\_versions/Match78/n3/match78n3\\_581-594.pdf](http://match.pmf.kg.ac.rs/electronic_versions/Match78/n3/match78n3_581-594.pdf). [COBISS.SI-ID [18108505](https://www.cobiss.si/id/18108505)]

VEGI KALAMAR, Alen, ŽERAK, Tadej, BOKAL, Drago. Counting Hamiltonian cycles in 2-tiled graphs. *Mathematics*. 2021, vol. 9, iss. 6, str. 1-27. ISSN 2227-7390. DOI: [10.3390/math9060693](https://doi.org/10.3390/math9060693). [COBISS.SI-ID [61574403](https://www.cobiss.si/id/61574403)]