UČNI NAČRT PREDMETA / COURSE SYLLABUS

Predmet:	Matematično modeliranje
Course title:	Mathematical Modelling

Študijski program in stopnja	Študijska smer	Letnik	Semester
Study programme and level	Study field	Academic year	Semester
Enovit magistrski študijski program druge stopnje Predmetni učitelj	/	**4.**	**8.**
Five-year master's degree program Subject Teacher	/		

Vrsta predmeta / Course type Obvezni / Compulsory

Univerzitetna koda predmeta / University course code:

Predavanja	Seminar	Sem. vaje	Lab. vaje	Teren. vaje	Samost. delo	ECTS
Lectures	Seminar	Tutorial	Laboratory work	Field work	Individ. work	
30	15		15		90	5

Nosilec predmeta / Lecturer: Drago Bokal

Jeziki /	**Predavanja / Lectures:**	SLOVENSKO/SLOVENE
Languages:	**Vaje / Tutorial:**	SLOVENSKO/SLOVENE

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Prerequisits:

Poznavanje osnov linearne algebre in vektorske analize.	Knowledge of basic linear algebra and calculus.

Vsebina:	**Content (Syllabus outline):**

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav:

- Pregled vrst matematičnih modelov. Proces izdelave matematičnega modela. Vrste spremenljivk.
- Matematično modeliranje in inovacijski proces.
- Modeliranje odločitev, odločitveno drevo, modelsko drevo, drevo igre, acikličen odločitven proces.
- Uvod v teorijo iger. Nashevo ravnovesje. Matrične igre z ničelno vsoto. Optimizacijski modeli s centraliziranim odločanjem, modeli teorije iger s porazdeljenim odločanjem.
- Linearni program. Dual. Farkaseva lema. Senčne cene. Analiza občutljivosti.
- Simulacijski modeli. Modeliranje sprememb z diferenčnimi in diferencialnimi enačbami.
- Matematično obnašanje dinamičnih sistemov.
- Analiza podatkov, verjetnost, Monte Carlo simulacija.

V okviru obvezne vsebine študentje izdelajo seminarsko nalogo, z dvema vmesnima in zaključno predstavitvijo, preko katerih utrdijo poznavanje procesa matematičnega modeliranja. Naloge so povezane z njihovo ali njihovih učencev bodočo kariero (izzivi iz upravljanja šole in vodenja pouka, zanimive igre, s katerimi se srečajo učenci, praktični problemi iz gospodarstva, teoretični problemi iz teorije modeliranja, optimizacije, algoritmov). Preostala predavanja se prilagodijo projektom, ki so jih izbrali študentje, in obsegajo naslednje vsebine:

- Uporaba matematičnih modelov v znanosti in pri učenju.
- Relacijski modeli na primeru urnika in razvrščanja nalog.
- Deterministični, stohastični, robustni problemi. Stohastični linearni program (diskretna spremenljivka). Dekompozicija. Deterministični in stohastični modeli optimizacije portfelja.
- Problem prehrane. Simpleksna metoda.
- Aplikacije teorije iger: optimalna strategija na tržišču z dvema konkurentoma.
- Čakalne vrste.
- Druge vsebine s področja matematičnega modeliranja, povezane s študentskimi projekti.

V okviru vsebin so predstavljene tudi odprtokodne in komercialne tehnološke rešitve za obravnavo navedenih modelov. Za organizacijo, predstavitev in analizo podatkov se uporablja Excel. Linearni programi se rešujejo z reševalci v tehnologiji AMPL, Matlab, R ali

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Mandatory content that familiarizes the students with fundamentals of mathematical modeling:

- Overview of mathematical model types. Process of mathematical model creation. Variable types.
- Mathematical model and inovation process.
- Decision modeling. Decision tree. Regression tree, game tree, acyclic decision process.
- Introduction to game theory. Nash equilibria. Matrix zero sum games. Optimization models with centralized decision making. Game theory models with distributed decision making.
- Linear program and its dual. Farkash lema. Shadow prices. Sensitivity analysis.
- Simulation models. Modeling changes with diference and diferential equations.
- Mathematical behaviour of dynamic systems.
- Data analysis, probability, monte carlo simulations.

Within the coursework, the students select problems whose result is a coursework report that is presented in two intermediate and one final presentation in front of the class. The problems are related to their or their pupil's future career (challenges in school management or class teaching, interesting games pupils encounter, practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list:

- Applications of mathematical models in science and learning.
- Relational models applied to timetabling and scheduling problems.
- Deterministic, stohastic, robust problems. Stochastic linear program (discrete variable). Decomposition. Deterministic and stochastic models of portfolio optimization.
- Diet problem. Simplex method.
- Applications of game theory: optimal strategy in two competitor market.
- Queues.
- Other material from the field of mathematical modeling, related to students' projects.

The students are familiarized with open-source and commercial technological solutions for treatment of the studied mathematical models. Excel is used for initial data organization, presentation, and analysis, Students are introduced to different linear programming solvers: AMPL,

Python-a, odvisno od okolja, iz katerega izhaja študentov problem. .	Matlab, R or Python, depending on the environment the student's problem is coming from.

Temeljni literatura in viri / Readings:

Osnovno / basic:

● Osais, Yahya Esmail. *Computer Simulation: A Foundational Approach Using Python*. Chapman and Hall/CRC, 2017.

● R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.

● J. Franklin, Methods of Mathematical Economics: Linear and Nonlinear Programming, Fixed-Point Theorems. Classics in Applied Mathematics 37, SIAM, 2002.

● Dossey, Giordano, McCrone, Weir, Mathematics Methods and Modelling for today's Mathematics Classroom, Brooks/Cole, Pacific Grove, 2002.

● Dossey, Silva (ur.), Sirnik, Mateja (ur.), Žakelj, Amalija: Matematika, Posodobitve pouka v gimnazijski praksi. 1. izd. Ljubljana: Zavod RS za šolstvo, 2010.

Dodatno / additional:

● E. Zakrajšek, Matematično modeliranje, DMFA – Založništvo, Ljubljana, 2004.

● J.D. Murray, Mathematical biology I. An introduction, Springer,New York, 2002.

● G. Polya, Kako rešujemo matematične probleme, DMFA, 1989.

Cilji in kompetence:

● Spoznati osnovne tehnike in prijeme matematičnega modeliranja po principih snovalskega razmišljanja.

● Spoznati teoretična ozadja matematičnega modeliranja in vpetost v druge znanstvene discipline.

● Uporabiti osnovne algoritme in hevristike za reševanje matematičnih problemov.

● Najti in uporabiti tehnološka orodja za izdelavo in raziskovanje matematičnega modela.

● Uporabiti znanje drugih matematičnih predmetov pri analizi praktičnih problemov.

● Pridobiti izkušnje pri izdelavi matematičnega modela, uporabnega pri kasnejši karieri.

● Pridobiti kompetenco iskanja in povzemanja literature o problemih, ki jih srečamo pri obravnavi modela.

● Seznaniti se z načini prepoznavanja za model pomembnih podatkov o problemu.

Objectives and competences:

● Understanding of basic techniques of mathematical modeling using principles of design thinking.

● Acquaintance with the theoretical background of mathematical modeling.

● Find and apply technological tools to develop and investigate a mathematical model.

● Understanding of basic applications of algorithms and heuristics to solve mathematical problems.

● Apply the knowledge from other mathematical areas in analysis of practical problems.

● Gain experience in developing a mathematical model, useful in their future career.

● Learn about sources of bibliography on problems related to studying mathematical models.

● Learn to distinguish the relevant data for the model under study.

● Pridobiti izkušnje pri pojasnjevanju matematičnega modela in zagovarjanju njegovih predpostavk.	● Gain experience in explaining and presenting the mathematical model and defending its assumptions.

Predvideni študijski rezultati:

Intended learning outcomes:

Znanje in razumevanje: ● Usvojenost matematičnih znanj, potrebnih za izdelavo in obravnavo matematičnih modelov po pristopih snovalskega razmišljanja. ● Usvojenost didaktičnih znanj, potrebnih za predstavitev matematičnih modelov, ki so predstavljena med Vsebinami in Cilji. ● Poznavanje matematičnih modelov, s katerimi se učitelj matematike pri pouku najpogosteje sreča in tehnik za njihovo obravnavo. Prenesljive/ključne spretnosti in drugi atributi: ● Pridobljena znanja in spretnosti, ki so navedene med Vsebinami in Cilji, so podlaga za uspešno soočanje z matematičnimi modeli, ki jih učitelji srečajo tekom izvajanja pedagoške prakse. ● Pridobljena spretnost povezovanja abstraktnega matematičnega znanja s primeri iz okolja, v katerem učitelj poučuje. ● Pridobljena spretnost motiviranja poglabljanja abstraktnega znanja s primeri uporabe teh znanj pri praktičnih problemih. ● Pridobljena spretnost uporabe sodobnih modelirnih orodij in tehnologij za namen študija matematičnih modelov.	Knowledge and Understanding: ● Adoption of special mathematical knowledge needed for developing and studying mathematical models, as presented in rubrics Contents and Objectives. ● Adoption of didactic knowledge and experience needed for presenting mathematical models, as presented in rubrics Contents and Objectives. ● Understanding basic mathematical models that a teacher of mathematics most commonly meets while teaching mathematics, as well as techniques for their studying. Transferable/Key Skills and other attributes: ● Adopted knowledge and skills, presented in the rubrics Contents and Objectives, are the basis for successful treatment of mathematical models that the teachers meet during teaching practice. ● Adopted the skill of connecting abstract mathematical knowledge with examples from the environment in which the teacher is teaching. ● Adopted the skill of motivating for deepening the understanding of abstract mathematical knowledge by applying this knowledge to practical problems. ● Adopted the skill of using modern modeling tools and technologies to study mathematical models.

Metode poučevanja in učenja:

Learning and teaching methods:

● Na predavanjih študentje spoznajo predpisano snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti. ● V okviru seminarskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja delajo na izbranih projektih. ● V okviru seminarja študentje predstavijo rezultate projektov, s čimer se priučijo suverenega javnega nastopanja in zagovarjanja svojih rezultatov.	● At the lectures, the students are familiarized with the required contents of the course. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course. ● Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning on three smaller projects. ● At the seminar, the students present the results of their projects, thus acquiring confidence with public presentation and defending their results.

Načini ocenjevanja:

Assessment:

Način (ustno izpraševanje, seminarska naloga - projekt)	Delež (v %) / Weight (in %)	Type (oral examination, coursework - project):
Seminarska naloga s tremi predstavitvami, ca. 90 ur samostojnega dela.	75 %	Coursework report, approx. 90 hours of individual work.
Ustni izpit	25 %	Oral exam
Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.		Each of the mentioned commitments must be assessed with a passing grade.
Pozitivna ocena pri seminarski nalogi je pogoj za pristop k izpitu.		Passing grade of the coursework report is required for taking the exam.
Reference nosilca / Lecturer's references:		

Glej COBISS/SICRIS.

http://sicris.izum.si/search/rsr.aspx?lang=slv&id=15413

BOKAL, Drago, STEINBACHER, Mitja. Phases of psychological optimal learning experience : task-based time allocation model. Central European Journal of Operations Research, ISSN 1435-246X, 2019, str. 1-23, doi: 10.1007/s10100-019-00609-0. [COBISS.SI-ID 24408328]

BOKAL, Drago, OPOROWSKI, Bogdan, RICHTER, Bruce, SALAZAR, Gelasio. Characterizing 2-crossing-critical graphs. Advances in applied mathematics, ISSN 0196-8858, 2016, vol. 74, str. 23-208. http://dx.doi.org/10.1016/j.aam.2015.10.003. [COBISS.SI-ID 17611353]

BOKAL, Drago, BREŠAR, Boštjan, JEREBIC, Janja. A generalization of Hungarian method and Hall's theorem with applications in wireless sensor networks. Discrete appl. math.. [Print ed.], 2012, vol. 160, iss. 4-5, str. 460-470. http://dx.doi.org/10.1016/j.dam.2011.11.007. [COBISS.SI-ID 16191577],

BOKAL, Drago, DEVOS, Matt, KLAVŽAR, Sandi, MIMOTO, Aki, MOOERS, Arne O. Computing quadratic entropy in evolutionary trees. Comput. math. appl. (1987). [Print ed.], 2011, vol. 62, no. 10, str. 3821-3828. http://dx.doi.org/10.1016/j.camwa.2011.09.030. [COBISS.SI-ID 16059481],

BOKAL, Drago, CHIMANI, Markus, LEANÕS, Jesús. Crossing number additivity over edge cuts. European journal of combinatorics, ISSN 0195-6698, 2013, vol. 34, iss. 6, str. 1010-1018. http://dx.doi.org/10.1016/j.ejc.2013.02.002. [COBISS.SI-ID 16624473]