



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>	<b>Verjetnost</b>					
<b>Course title:</b>	Probability					
<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field			<b>Letnik</b> Academic year	<b>Semester</b> Semester	
Matematika				<b>3.</b>	<b>5.</b>	
Mathematics				<b>3<sup>rd</sup></b>	<b>5<sup>th</sup></b>	
<b>Vrsta predmeta / Course type</b>				obvezni/compulsory		
<b>Univerzitetna koda predmeta / University course code:</b>						
<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Sem. vaje</b> Tutorial	<b>Lab. vaje</b> Laboratory work	<b>Teren. vaje</b> Field work	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
<b>60</b>		<b>45</b>			<b>135</b>	<b>8</b>
<b>Nosilec predmeta / Lecturer:</b>				Dominik BENKOVIČ		
<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>		SLOVENSKO/SLOVENE			
	<b>Vaje / Tutorial:</b>		SLOVENSKO/SLOVENE			
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>				<b>Prerequisites:</b>		
Vsaka izmed naštetih obveznosti v načinih ocenjevanja mora biti opravljena s pozitivno oceno. Pozitivna ocena pri pisnem izpitu je pogoj za pristop k ustnemu izpitu.				Each of the mentioned commitments must be assessed with a passing grade. Passing grade of the written exam is required for taking the oral exam.		
<b>Vsebina:</b>				<b>Content (Syllabus outline):</b>		
<ul style="list-style-type: none"><li>Osnovni pojmi verjetnosti: Poskus, dogodek in algebra dogodkov. Klasična, statistična, geometrijska in aksiomatična definicija verjetnosti. Osnovne lastnosti verjetnosti.</li><li>Pogojna verjetnost: Neodvisni dogodki. Relejni poskusi, formula za popolno verjetnost in Bayesova formula.</li></ul>				<ul style="list-style-type: none"><li>Basic concepts of probability: experiment, event and sample space. The classical, statistical, geometrical and axiomatic definition of probability. Basic properties of probability.</li><li>Conditional probability: Independent events. The formula of total probability and the Bayes' rule.</li></ul>		

<ul style="list-style-type: none"> <li>• Zaporedja neodvisnih poskusov: Bernoullijevo zaporedje poskusov. Binomska porazdelitev. Limitni izreki: Poissonova formula, Laplaceova lokalna in integralska formula. Bernoullijev zakon velikih števil.</li> <li>• Slučajne spremenljivke: Porazdelitvena funkcija in njene osnovne lastnosti. Diskretne in zvezne porazdelitve. Pomembne porazdelitve. Funkcije slučajnih spremenljivk.</li> <li>• Številске karakteristike slučajnih spremenljivk: Matematično upanje in disperzija. Višji momenti in vrstilne karakteristike.</li> <li>• Slučajni vektorji. Diskretni in zvezni slučajni vektorji. Neodvisnost slučajnih spremenljivk. Funkcije slučajnih vektorjev. Kovarianca in korelacijski koeficient.</li> <li>• Rodovne in karakteristične funkcije: Definicija in osnovne lastnosti rodovnih in karakterističnih funkcij.</li> <li>• Limitni izreki teorije verjetnosti: Zakon velikih števil. Centralni limitni izrek.</li> <li>• Uvod v teorijo slučajnih procesov: Markovske verige. Klasifikacija stanj. Stacionarna porazdelitev. Primeri: slučajni sprehod, proces razvejanja, proces rojevanja, Poissonov proces.</li> </ul>	<ul style="list-style-type: none"> <li>• Sequences of independent trials: Bernoulli trials. The binomial distribution. Limit theorems: Poisson's theorem, local and integral Laplace theorems. The Bernoulli's law of large numbers.</li> <li>• Random variables: The distribution function and its basic properties. Discrete and continuous distributions. Examples of most important distributions. Functions of random variables.</li> <li>• Numerical characteristics of random variables: Mathematical expectation and variance. Higher moments and order characteristics.</li> <li>• Random vectors: Discrete and continuous random vectors. Independence of random variables. Functions of random vectors. Covariance and correlation coefficient.</li> <li>• Generating and characteristic functions: Definition and elementary properties of generating and characteristic functions.</li> <li>• Limit theorems of probability theory: Law of large numbers. The central limit theorem.</li> <li>• Introduction to random processes: Markov chains. Classification of states. Stationary distribution. Examples: random walk, branching process, birth process, Poisson process.</li> </ul>
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#### **Temeljni literatura in viri / Readings:**

1. D. Benkovič: Verjetnost, Univerza v Mariboru, Univerzitetna založba, Maribor 2022.
2. R. Jamnik: Verjetnostni račun in statistika, DMFA, Ljubljana 1995.

#### **Dodatna literatura / Additional Readings:**

3. R. Drnovšek, T. Košir, E. Kramar, G. Lešnjak: Zbirka rešenih nalog iz verjetnostnega računa, DMFA, Ljubljana 1998.
4. B. V. Gnedenko: The theory of probability, Mir Publishers, Moskva 1988.
5. G. Grimmett, D. Stirzaker: Probability and random processes, Oxford University Press, Oxford 2009.

6. M. Hladnik: Verjetnost in statistika, Fakulteta za računalništvo in informatiko, Ljubljana 2002.
7. J. Pitman: Probability, Springer-Verlag, New York [etc.] 1993.
8. D. Williams: Weighing the odds : a course in probability and statistics, Cambridge University Press, Cambridge 2004.

### Cilji in kompetence:

Glavni cilj predmeta je proučiti najpomembnejše koncepte in rezultate teorije verjetnosti.

### Objectives and competences:

The main goal of the course is to study the fundamental concepts and results of probability theory.

### Predvideni študijski rezultati:

Znanje in razumevanje:

- Razumevanje verjetnosti in različnih pristopov k definiranju le-te ter osvojitve različnih tehnik računanja verjetnosti.
- Osvojiti najpreprostejši primer slučajnega procesa - homogene markovske verige.
- Razumevanje in poznavanje osnovnih rezultatov teorije verjetnosti, ki so povezani s slučajnimi spremenljivkami in vektorji.
- Poznavanje osnovnih rezultatov, ki so povezani z rodovnimi in karakterističnimi funkcijami ter limitnimi izreki.

Prenesljive/ključne spretnosti in drugi atributi:

- Uporaba znanja iz teorije verjetnosti pri statistiki in na drugih področjih uporabne matematike.

### Intended learning outcomes:

Knowledge and Understanding:

- Understanding the notion of probability, different approaches to its definition, and techniques of calculating probability.
- Understanding of the simplest example of the random process – Markov chain.
- Understanding and knowledge of basic results of the probability theory which are related to random variables and vectors.
- Knowledge of basic results which are related to generating and characteristic functions and also to limit theorems.

Transferable/Key Skills and other attributes:

- Knowledge transfer of methods of probability theory into statistics and to other fields of applied mathematics.

### Metode poučevanja in učenja:

- Predavanja
- Teoretične vaje

### Learning and teaching methods:

- Lectures
- Theoretical exercises

### Načini ocenjevanja:

### Assessment:

	Delež (v %) / Weight (in %)	
Pisni izpit	50%	Written exam
Ustni izpit	50%	Oral exam

### Opombe:

Pisni izpit se lahko nadomesti s kolokviji v enakem deležu 50%.

**Comments:**

Written exam can be replaced by written midterm examination in the weight of 50%.

**Reference nosilca / Lecturer's references:**

1. BENKOVIČ, Dominik, EREMITA, Daniel. Generalized derivations of current Lie algebras. *Communications in algebra*. 2024, vol. 52, iss. 11, str. 4603-4611. ISSN 0092-7872. <https://www.tandfonline.com/doi/full/10.1080/00927872.2024.2354423>, DOI: [10.1080/00927872.2024.2354423](https://doi.org/10.1080/00927872.2024.2354423). [COBISS.SI-ID [200554755](https://www.cobiss.si/id/200554755)]
2. BENKOVIČ, Dominik. A note on generalized Jordan n-derivations of unital rings. *Indian Journal of pure and applied mathematics*. 2024, vol. 55, str. 623-627. ISSN 0019-5588. <https://doi.org/10.1007/s13226-023-00394-2>, DOI: [10.1007/s13226-023-00394-2](https://doi.org/10.1007/s13226-023-00394-2). [COBISS.SI-ID [224333827](https://www.cobiss.si/id/224333827)]
3. BENKOVIČ, Dominik. Lie  $\sigma$ -derivations of triangular algebras. *Linear and Multilinear Algebra*. 2022, vol. 70, iss. 15, str. 2966-2983. ISSN 0308-1087. <https://www.tandfonline.com/doi/full/10.1080/03081087.2020.1820431>, DOI: [10.1080/03081087.2020.1820431](https://doi.org/10.1080/03081087.2020.1820431). [COBISS.SI-ID [127110659](https://www.cobiss.si/id/127110659)], [JCR]
4. BENKOVIČ, Dominik, GRAŠIČ, Mateja. Jordan  $\{g,h\}$ -derivations of unital algebras. *Operators and matrices*. 2022, vol. 16, no. 2, str. 415-428. ISSN 1846-3886. <http://oam.ele-math.com/16-32/Jordan-g,h-derivations-of-unital-algebras>, DOI: [10.7153/oam-2022-16-32](https://doi.org/10.7153/oam-2022-16-32). [COBISS.SI-ID [114972163](https://www.cobiss.si/id/114972163)], [JCR]
5. BENKOVIČ, Dominik. Generalized Lie n-derivations of triangular algebras. *Communications in algebra*. 2019, vol. 47, iss. 12, str. 5294-5302. ISSN 0092-7872. <https://doi.org/10.1080/00927872.2019.1617875>, DOI: [10.1080/00927872.2019.1617875](https://doi.org/10.1080/00927872.2019.1617875). [COBISS.SI-ID [18879833](https://www.cobiss.si/id/18879833)], [JCR]