UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	**Matematično modeliranje**
Course title:	Mathematical Modelling

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika		**3.**	**6.**
Mathematics		**3.**	**6.**

Vrsta predmeta / Course type izbirni/elective

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	**15**	**-**	**15**	**-**	**120**	**6**

Nosilec predmeta / Lecturer: Drago Bokal

Jeziki / Languages:	Predavanja / Lectures:	Slovensko/Slovene
	Vaje / Tutorial:	Slovensko/Slovene

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	Prerequisits:
Poznavanje enostavnih algoritmov. Poznavanje osnov linearne algebre in vektorske analize. Vsaka izmed naštetih obveznosti v načinih ocenjevanja mora biti opravljena s pozitivno oceno. Pozitivna ocena izdelka je pogoj za pristop k ustnemu izpitu.	Knowledge of simple algorithms. Knowledge of basic linear algebra and calculus. Each of the mentioned commitments must be assessed with a passing grade. Passing grade of the output is required for taking the oral exam.

Vsebina:	Content (Syllabus outline):
Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav:	Mandatory content that familiarizes the students with fundamentals of mathematical modeling:

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav:

- Pregled vrst matematičnih modelov. Proces izdelave matematičnega modela. Vrste spremenljivk.
- Matematično modeliranje in inovacijski proces.
- Matematični modeli znanja in učenja: spodbujevano učenje, vmesniška teorija zaznav, univerzalni model procesa.
- Modeliranje procesov. Sekvenčni diagrami, diagrami poteka, Petrijeve mreže.
- Podatkovno modeliranje. Entitetni diagram, diagram razredov.
- Modeliranje odločitev, odločitveno drevo, modelsko drevo, drevo igre, acikličen odločitven proces.
- Linearni program. Dual. Farkaseva lema. Senčne cene. Analiza občutljivosti.
- Analiza podatkov, verjetnost, Monte Carlo simulacija.

V okviru obvezne vsebine študentje izdelajo seminarsko nalogo, z dvema vmesnima in zaključno predstavitvijo, preko katerih utrdijo poznavanje procesa matematičnega modeliranja. Naloge so povezane z njihovo bodočo kariero (praktični problemi iz gospodarstva, teoretični problemi iz teorije modeliranja, optimizacije, algoritmov). Preostala predavanja se prilagodijo projektom, ki so jih izbrali študentje, in obsegajo naslednje vsebine:

- Uvod v teorijo iger. Nashevo ravnovesje. Matrične igre z ničelno vsoto. Optimizacijski modeli s centraliziranim odločanjem, modeli teorije iger s porazdeljenim odločanjem.
- Simulacijski modeli. Modeliranje sprememb z diferenčnimi in diferencialnimi enačbami.
- Matematično obnašanje dinamičnih sistemov.
- Uporaba matematičnih modelov v znanosti in tehniki.

Mandatory content that familiarizes the students with fundamentals of mathematical modeling:

- Overview of mathematical model types. Process of mathematical model creation. Variable types.
- Mathematical model and inovation process.
- Mathematical models of knowledge and learning: reinforcement learning, interface theory of perception, universal process model.
- Process modelling. Sequence diagrams, flow charts, Petri nets.
- Data modelling. Entity relationship diagrams, Class diagrams.
- Decision modeling. Decision tree. Regression tree, game tree, acyclic decision process.
- Linear program and its dual. Farkash lema. Shadow prices. Sensitivity analysis.
- Data analysis, probability, Monte Carlo simulations.

Within the coursework, the students select problems whose result is a coursework report that is presented in two intermediate and one final presentation in front of the class. The problems are related to their future career (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list:

- Introduction to game theory. Nash equilibria. Matrix zero sum games. Optimization models with centralized decision making. Game theory models with distributed decision making.
- Simulation models. Modeling changes with diference and diferential equations.
- Mathematical behaviour of dynamic systems.
- Applications of mathematical models in science and engineering.

- Relacijski modeli na primeru urnika in razvrščanja nalog. - Deterministični, stohastični, robustni problemi. Stohastični linearni program (diskretna spremenljivka). Dekompozicija. Deterministični in stohastični modeli optimizacije portfelja. - Problem prehrane. Simpleksna metoda. - Aplikacije teorije iger: optimalna strategija na tržišču z dvema konkurentoma. - Čakalne vrste. - Druge vsebine s področja matematičnega modeliranja, povezane s študentskimi projekti. V okviru vsebin so predstavljene tudi odprtokodne in komercialne tehnološke rešitve za obravnavo navedenih modelov. Za organizacijo, predstavitev in analizo podatkov se uporablja Excel. Linearni programi se rešujejo z reševalci v tehnologiji Python, R, AMPL, Matlab, odvisno od okolja, iz katerega izhaja študentov problem.	- Relational models applied to timetabling and scheduling problems. - Deterministic, stohastic, robust problems. Stohastic linear program (discrete variable). Decomposition. Deterministic and stohastic models of portfolio optimization. - Diet problem. Simplex method. - Applications of game theory: optimal strategy in two competitor market. - Queues. - Other material from the field of mathematical modeling, related to students' projects. The students are familiarized with open-source and commercial technological solutions for treatment of the studied mathematical models. Excel is used for initial data organization, presentation, and analysis. Students are introduced to different linear programming solvers: Python, R, AMPL, Matlab, depending on the environment the student's problem is coming from.

Temeljni literatura in viri / Readings:

● Osais, Yahya Esmail. *Computer Simulation: A Foundational Approach Using Python*. Chapman and Hall/CRC, 2017.
● R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.
● J. Franklin, Methods of Mathematical Economics: Linear and Nonlinear Programming, Fixed-Point Theorems. Classics in Applied Mathematics 37, SIAM, 2002.
● Dossey, Giordano, McCrone, Weir, Mathematics Methods and Modelling for today's Mathematics Classroom, Brooks/Cole, Pacific Grove, 2002.
● E. Zakrajšek, *Matematično modeliranje,* DMFA – Založništvo, Ljubljana, 2004.
Dodatno / additional:
● J.D. Murray, *Mathematical biology I. An introduction*, Springer,New York, 2002.
● G. Polya, *Kako rešujemo matematične probleme*, DMFA, 1989.

Cilji in kompetence:	**Objectives and competences:**
● Spoznati osnovne tehnike in prijeme matematičnega modeliranja po principih snovalskega razmišljanja. ● Spoznati teoretična ozadja matematičnega modeliranja in vpetost v druge znanstvene discipline.	● Understanding of basic techniques of mathematical modeling using principles of design thinking. ● Acquaintance with the theoretical background of mathematical modeling. ● Find and apply technological tools to develop and investigate a mathematical model.

● Uporabiti osnovne algoritme in hevristike za reševanje matematičnih problemov.
● Najti in uporabiti tehnološka orodja za izdelavo in raziskovanje matematičnega modela.
● Uporabiti znanje drugih matematičnih predmetov pri analizi praktičnih problemov.
● Pridobiti izkušnje pri izdelavi matematičnega modela, uporabnega pri kasnejši karieri.
● Pridobiti kompetenco iskanja in povzemanja literature o problemih, ki jih srečamo pri obravnavi modela.
● Seznaniti se z načini prepoznavanja za model pomembnih podatkov o problemu.
● Pridobiti izkušnje pri pojasnjevanju matematičnega modela in zagovarjanju njegovih predpostavk.
● Spoznati Okvir digitalnih kompetenc za državljane 2.2 in Evropski kompetenčni okvir za trajnostnost kot primer učnih prostorov.
● Prepoznati bistvene kompetence v navedenih kompetenčnih okvirih in samooceniti svoje znanje.
●Zaznati potrebo po nadgradnji izbranih kompetenc omenjenih okvirov in pridobiti informacije za njihovo nadgrajevanje.

● Understanding of basic applications of algorithms and heuristics to solve mathematical problems.
● Apply the knowledge from other mathematical areas in analysis of practical problems.
● Gain experience in developing a mathematical model, useful in their future career.
● Learn about sources of bibliography on problems related to studying mathematical models.
● Learn to distinguish the relevant data for the model under study.
● Gain experience in explaining and presenting the mathematical model and defending its assumptions.
●Learn about The Digital Competence Framework For Citizens and GreenComp competence framework as examples of learning spaces.od
●Recognize most relevant competences in the above competence frameworks and self-evaluate their knowledge of these competences.nd
●Recognize the need for upgrading the selected competences of these frameworks and obtain relevant information for their upgrading.

Predvideni študijski rezultati:

Znanje in razumevanje:

● Poglobiti znanje iz matematičnih metod optimizacije.
● Poglobiti znanje iz uporabe matematičnega modeliranja v znanosti in praksi.
● Podrobno poglobi znanje iz zahtevnejših aplikacij matematičnega modeliranja v finančni optimizaciji.

Prenesljive/ključne spretnosti in drugi atributi:
● Direktne aplikacije v finančni matematiki, ekonomiji, poslovnih vedah, inžinirstvu, kemiji in številnih drugih družboslovnih in naravoslovnih vedah. Obenem principi

Intended learning outcomes:

Knowledge and Understanding:

● To deepen the knowledge of mathematical methods of optimization.
● To deepen the knowledge of applications of mathematical modelling in research and practice.
● To deepen the knowledge of details of advanced applications of mathematical modelling in financial optimization.

Transferable/Key Skills and other attributes:
● Direct applications in finacial mathematics, economy, business, engineering, chemistry, and numerous other social and natural sciences. Also, principles of linear

linearne optimizacije tvorijo osnovo za matematično programiranje. ● Izdelava temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.	optimization are foundations for mathematical programming. ● Preparation of a detailed technical report or focused report paper describing a mathematical model of a specific mathematical problem the students encounter while investigation their possible future careers.

Metode poučevanja in učenja: / **Learning and teaching methods:**

● Na predavanjih študentje spoznajo predpisano snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti. ● V okviru seminarskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja delajo na izbranih projektih. ● V okviru seminarja študentje uporabljajo, analizirajo in vrednotijo svoje projekte za potrebe ustvarjanja novih rešitev za okolje, iz katerega izvirajo problemi. ● Z uporabo izkušanja matematičnega modeliranja študenti merijo svoj napredek pri predmetu, ga reflektirajo in tako uporabljajo njegov matematični model za opazovanje izboljšanja, ki izvira iz matematično podprtega odločanja. ● Tri predstavitve njihovih rezultatov pomagajo študentom priučiti se suverenosti javnega nastopanja in zagovarjanja doseženih rezultatov. Osredotočeni dvodnevni organizirani hackathon pomaga študentom izkusiti veselje ob osredotočenem raziskovanju in sodelovanju z vrstniki.	● At the lectures, the students are familiarized with the required contents of the course. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course. ● Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning on three smaller projects. ● At the seminar, the students apply, analyse, and evaluate their projects in order to create new solutions desired by the environment the problems are coming from. ● Applying embedded mathematical modelling, the students measure and reflect upon their progress thus using its mathematical model to experience improvement coming from mathematically supported decision making. ● Three presentations of their results help students acquire confidence with public presentation and defending their results. ● Focused two-day workshop-style organized hackathon helps students experience the joy of focused research and collaboration with peers.

Načini ocenjevanja: / **Assessment:**

Način (ustno izpraševanje, seminarska naloga - projekt):	Delež (v %) / Weight (in %)	Type (oral examination, coursework - project):
Izdelek	75%	Output
Ustni izpit	25%	Oral exam

Reference nosilca / Lecturer's references:

Glej COBISS/SICRIS. http://sicris.izum.si/search/rsr.aspx?lang=slv&id=15413
1. BOKAL, Drago, CHIMANI, Markus, NOVER, Alexander, SCHIERBAUM, Jöran, STOLZMANN, Tobias, WAGNER, Mirko H., WIEDERA, Tilo. Properties of large 2-crossing-critical graphs. *Journal of graph algorithms and applications*. 2022, vol. 26, no. 1, str. 111-147. ISSN 1526-1719. DOI: 10.7155/jgaa.00585. [COBISS.SI-ID 144719363],
financer: ARRS, Projekti, J1-8130, SI, Prekrižna števila in njihova uporaba; ARRS, Programi, P1-0297, SI, Teorija grafov; Evropski socialni sklad, SI, EU, Inovativno učenje in poučevanje v visokem šolstvu, INOVUP
2. SMOLE, Andreja, JAGRIČ, Timotej, BOKAL, Drago. Principal/Two-Agent model with internal signal. *Central European journal of operations research*. sep. 2021, vol. 29, no. 3, str. 791-808, ilustr. ISSN 1435-246X. https://link.springer.com/content/pdf/10.1007/s10100-020-00719-0.pdf, DOI: 10.1007/s10100-020-00719-0. [COBISS.SI-ID 75068163],
3. BOKAL, Drago, JEREBIC, Janja. Guarding a subgraph as a tool in pursuit-evasion games. *Discussiones mathematicae. Graph theory*. 2022, vol. 42, no. 1, str. 123-138. ISSN 1234-3099. https://www.dmgt.uz.zgora.pl/publish/view_vol_article.php?E8F29296C3FB4B933A2E. [COBISS.SI-ID 8147219],
4. VEGI KALAMAR, Alen, ŽERAK, Tadej, BOKAL, Drago. Counting Hamiltonian cycles in 2-tiled graphs. *Mathematics*. 2021, vol. 9, iss. 6, str. 1-27. ISSN 2227-7390. DOI: 10.3390/math9060693. [COBISS.SI-ID 61574403],
5. BOKAL, Drago, STEINBACHER, Mitja. Phases of psychologically optimal learning experience : task-based time allocation model. *Central European journal of operations research*. Sep. 2019, vol. 27, iss. 3, str. 863-885. ISSN 1435-246X. DOI: 10.1007/s10100-019-00609-0. [COBISS.SI-ID 24408328],