



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

| UČNI NAČRT PREDMETA / COURSE SYLLABUS  |                                      |                              |                                     |  |                                      |             |
|--|--------------------------------------|------------------------------|-------------------------------------|--|--------------------------------------|-------------|
| <b>Predmet:</b>  | <b>Analiza III</b>                   |                              |                                     |  |                                      |             |
| <b>Course title:</b>   | <b>Analysis III</b>                  |                              |                                     |  |                                      |             |
|  |                                      |                              |                                     |  |                                      |             |
| <b>Študijski program in stopnja</b><br>Study programme and level   | <b>Študijska smer</b><br>Study field |                              |                                     | <b>Letnik</b><br>Academic year   | <b>Semester</b><br>Semester          |             |
| Matematika   |                                      |                              |                                     | 2.   | 4.                                   |             |
| Mathematics  |                                      |                              |                                     | 2.   | 4.                                   |             |
| <b>Vrsta predmeta / Course type</b>  |                                      |                              |                                     | Obvezni / Obligatory   |                                      |             |
|  |                                      |                              |                                     |  |                                      |             |
| <b>Univerzitetna koda predmeta / University course code:</b>   |                                      |                              |                                     |  |                                      |             |
|  |                                      |                              |                                     |  |                                      |             |
| <b>Predavanja</b><br>Lectures  | <b>Seminar</b><br>Seminar            | <b>Sem. vaje</b><br>Tutorial | <b>Lab. vaje</b><br>Laboratory work | <b>Teren. vaje</b><br>Field work   | <b>Samost. delo</b><br>Individ. work | <b>ECTS</b> |
| 60   |                                      | 60                           |                                     |  | 150                                  | 9           |
| <b>Nosilec predmeta / Lecturer:</b>  |                                      |                              |                                     | Bojan HVALA  |                                      |             |
|  |                                      |                              |                                     |  |                                      |             |
| <b>Jeziki / Languages:</b>   | <b>Predavanja / Lectures:</b>        | SLOVENSKO/SLOVENE            |                                     |  |                                      |             |
|  | <b>Vaje / Tutorial:</b>              | SLOVENSKO/SLOVENE            |                                     |  |                                      |             |
| <b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>   |                                      |                              |                                     | <b>Prerequisites:</b>  |                                      |             |
| Jih ni,  |                                      |                              |                                     | There are none.  |                                      |             |
| <b>Vsebina:</b>  |                                      |                              |                                     | <b>Content (Syllabus outline):</b>   |                                      |             |
| Metrični prostori. Odprte in zaprte množice. Zaporedja v metričnih prostorih. Polnost. Zveznost in enakomerna zveznost. Kompaktnost in povezanost.   |                                      |                              |                                     | Metric spaces. Open and closed sets. Sequences in metric spaces. Complete metric spaces. Continuous and uniformly continuous functions. Compact and connected sets.          |                                      |             |
| Funkcije več realnih spremenljivk. Zveznost, parcialna odvedljivost. Višji parcialni odvodi. Taylorjeva formula. Lokalni in globalni ekstremi. Izrek o implicitni funkciji. Vežani ekstremi. |                                      |                              |                                     | Functions of several real variables. Partial derivatives, higher derivatives. Taylor's formula. Local and absolute extrema. Implicit function theorem. Lagrange multipliers. |                                      |             |
| Preslikave $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ , diferencial.   |                                      |                              |                                     | Mappings $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ , differentiability.   |                                      |             |
| Integral s parametrom, Eulerjevi funkciji beta in gama.  |                                      |                              |                                     | Euler, Gamma and beta functions.   |                                      |             |



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### Temeljni literatura in viri / Readings:

- J. Vrabec: *Metrični prostori*. Ljubljana: DMFA, 1993.
- D. Varberg, E. Purcell, S. Rigdon: *Calculus*. Prentice Hall, 2006.
- M. H. Protter, C. B. Morrey: *Intermediate calculus*. New York : Springer, 1985
- S. Lang: *Calculus of several variables*, Reading: Addison – Wesley, 1973.
- G. Fih tengol'c: *Osvovy matematičeskogo analiza I, II, III*. Moskva: Fizmatgiz, 1956.
- M. Dobovišek: *Rešene naloge iz analize II*. Ljubljana: DMFA, 1996.
- B. Hvala: *Zbirka izpitnih nalog iz analize*. Ljubljana: DMFA, 1996.

### Cilji in kompetence:

Predstaviti osnovne koncepte in rezultate metričnih prostorov. Na primerih utemeljiti potrebo po tej strukturi. Predstaviti ključna dejstva o funkcijah več spremenljivk s poudarkom na sistematičnem posploševanju rezultatov za funkcije ene spremenljivke. Uvod v integral s parametrom s poudarkom na Eulerjevih funkcija beta in gama.

### Objectives and competences:

Present the basic concepts and results on metric spaces. Justify the need for this structure using several examples. To present key facts about the functions of several variables with emphasis on systematic generalization of results for the functions of one variable. Introduction to integral with a parameter with an emphasis on Euler's beta and gamma functions.

### Predvideni študijski rezultati:

Znanje in razumevanje:

Po zaključku tega predmeta bo študent

- Obvladal osnovne koncepte metričnih prostorov in poznal ključne s tem povezane rezultate.
- Znal izračunati parcialne odvode in Jacobijevo matriko funkcije v določeni točki ter se zavedal njihovega pomena.
- Znal poiskati običajne in vezane ekstreme funkcij več spremenljivk.
- Znal pojasniti vsebinsko povezavo med lastnimi vrednostmi Hessejeve matrike in ekstremi ustrezne funkcije več spremenljivk.
- Znal uporabiti enega od izrekov o implicitni funkciji.
- Poznal osnovne rezultate v zvezi z integrali s parametrom in jih znal uporabiti v konkretnih situacijah.

### Intended learning outcomes:

Knowledge and Understanding:

On completion of this course a student will be able to

- Master the basic concepts of metric spaces and recall the key related results.
- Calculate the partial derivatives and the Jacobian matrix of a function at a given point and be aware of their significance.
- Solve unconstrained and constrained maxima/minima problems for functions of several variables.
- Explain the relationship between the Hessian matrix and the maxima/minima of the corresponding functions of several variables.
- Apply the implicit function theorems.
- Recall and apply the basic results concerning integrals with a parameter.
- Recognize the beta and gamma functions and master their standard uses.



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- Prepoznal funkciji beta in gama in obvladal njune standardne uporabe.

Prenesljive/ključne spretnosti in drugi atributi:

- *Reševanje problemov, kritično razmišljanje:* Kritična evalvacija rezultatov pri iskanju ekstremov funkcij več spremenljivk.
- *Sposobnost prilagajanja novim razmeram:* Posplošitev rezultatov ob prehodu s preprostejših množic, funkcij in struktur na zahtevnejše.

Transferable / Key Skills and other attributes:

- *Problem solving, critical thinking:* Critical evaluation of results when solving maxima/minima problems for functions of several variables.
- *Ability to adapt to new situations:* The generalization of results when moving from simple sets, functions and structures to more demanding ones.

To be aware of the importance of compactness, completeness and connectedness of metric spaces.

To solving extremum problems of different kinds.  
acquaintance with the concept of the integral with parameter and with Gamma and Beta functions.

**Metode poučevanja in učenja:**

- Predavanja
- Teoretične vaje

**Learning and teaching methods:**

- Lectures
- Theoretical exercises

**Načini ocenjevanja:**

**Assessment:**

Izpit:

Pisni izpit – problemi

Ustni izpit

Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.

Opravljen pisni izpit – problemi je pogoj za pristop k ustnemu izpitu.

Pisni izpit – problemi se lahko nadomesti z dvema delnima testoma

Delež (v %) /  
Weight (in %)

50%

50%

Exams:

Written exam – problems

Oral exam

Each of the mentioned assessments must be assessed with a passing grade.

Passing grade of written exam – problems is required to take the oral exam.

Written exam – problems can be replaced with two mid-term tests.



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(sprotne obveznosti).

**Reference nosilca / Lecturer's references:**

1. HVALA, Bojan. Cevian cousins of a triangle centroid. *Journal for geometry and graphics*, ISSN 1433-8157, 2015, vol. 19, no. 2, str. 211-218. [COBISS.SI-ID [22024200](#)]
2. HVALA, Bojan. A generalized Seebach's theorem. *Beiträge zur Algebra und Geometrie*, ISSN 0138-4821, 2014, vol. 55, iss. 2, str. 471-478, doi: [10.1007/s13366-013-0165-3](https://doi.org/10.1007/s13366-013-0165-3). [COBISS.SI-ID [20843272](#)]
3. HVALA, Bojan. Značilne točke trikotnika kot funkcije. *Obzornik za matematiko in fiziko*, ISSN 0473-7466, 2014, letn. 61, št. 1, str. 1-14. [COBISS.SI-ID 16937817].
4. HVALA, Bojan. Diophantine Steiner triples and Pythagorean-type triangles. *Forum geometricorum*, ISSN 1534-1178, 2010, vol. 10, str. 93-97. <http://forumgeom.fau.edu/FG2010volume10/FG201010.pdf>. [COBISS.SI-ID [15669337](#)]
5. HVALA, Bojan. Modernizing mathematics education in Slovenia : a teacher friendly approach. V: LAMANAUSKAS, Vincentas (ur.). *Challenges of science, mathematics and technology teacher education in Slovenia*, (Problems of education in the 21st century, ISSN 1822-7864, vol. 14). Siauliai: Scientific Methodological Center Scientia Educologica. 2009, str. 34-43. [COBISS.SI-ID [17351944](#)]