



Univerza v Mariboru

Fakulteta za naravoslovje
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS

Predmet:	Matematično modeliranje
Course title:	Mathematical Modelling

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika		2	4
Mathematics		2	4

Vrsta predmeta / Course type

Obvezni / Obligatory

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15		15		120	6

Nosilec predmeta / Lecturer:

Drago Bokal

Jeziki /

Languages:

Predavanja /

Lectures:

Slovensko/Slovene ali Angleško/English

Vaje / Tutorial:

Slovensko/Slovene ali Angleško/English

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Poznavanje enostavnih algoritmov.
Poznavanje osnov linearne algebre in vektorske analize.

Prerequisites:

Knowledge of simple algorithms.
Knowledge of basic linear algebra and calculus.

Vsebina:

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav:

- Pregled vrst matematičnih modelov. Proces izdelave matematičnega modela. Vrste spremenljivk.

Content (Syllabus outline):

Mandatory content that familiarizes the students with fundamentals of mathematical modeling:

- Overview of mathematical model types. Process of mathematical model creation. Variable types.

- Matematično modeliranje in inovacijski proces.
- Modeliranje odločitev, odločitveno drevo, modelsko drevo, drevo igre, acikličen odločitven proces.
- Uvod v teorijo iger. Nashevo ravnovesje. Matrične igre z ničelno vsoto. Optimizacijski modeli s centraliziranim odločanjem, modeli teorije iger s porazdeljenim odločanjem.
- Linearni program. Dual. Farkaseva lema. Senčne cene. Analiza občutljivosti.
- Simulacijski modeli. Modeliranje sprememb z diferenčnimi in diferencialnimi enačbami.
- Matematično obnašanje dinamičnih sistemov.
- Analiza podatkov, verjetnost, Monte Carlo simulacija.

V okviru obvezne vsebine študentje izdelajo seminarsko nalogo, z dvema vmesnima in zaključno predstavitvijo, preko katerih utrdijo poznavanje procesa matematičnega modeliranja. Naloge so povezane z njihovo bodočo kariero (praktični problemi iz gospodarstva, teoretični problemi iz teorije modeliranja, optimizacije, algoritmov). Preostala predavanja se prilagodijo projektu, ki so jih izbrali študentje, in obsegajo naslednje vsebine:

- Uporaba matematičnih modelov v znanosti in pri učenju.
- Relacijski modeli na primeru urnika in razvrščanja nalog.
- Deterministični, stohastični, robustni problemi. Stohastični linearni program (diskretna spremenljivka). Dekompozicija. Deterministični in stohastični modeli optimizacije portfelja.
- Problem prehrane. Simpleksna metoda.
- Aplikacije teorije iger: optimalna strategija na tržišču z dvema konkurentoma.
- Čakalne vrste.

- Mathematical model and innovation process.
- Decision modeling. Decision tree. Regression tree, game tree, acyclic decision process.
- Introduction to game theory. Nash equilibria. Matrix zero sum games. Optimization models with centralized decision making. Game theory models with distributed decision making.
- Linear program and its dual. Farkash lema. Shadow prices. Sensitivity analysis.
- Simulation models. Modeling changes with difference and differential equations.
- Mathematical behaviour of dynamic systems.
- Data analysis, probability, Monte Carlo simulations.

Within the coursework, the students select problems whose result is a coursework report that is presented in two intermediate and one final presentation in front of the class. The problems are related to their future career (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list:

- Applications of mathematical models in science and learning.
- Relational models applied to timetabling and scheduling problems.
- Deterministic, stochastic, robust problems. Stochastic linear program (discrete variable). Decomposition. Deterministic and stochastic models of portfolio optimization.
- Diet problem. Simplex method.
- Applications of game theory: optimal strategy in two competitor market.
- Queues.
- Other material from the field of mathematical modeling, related to students' projects.

<ul style="list-style-type: none"> • Druge vsebine s področja matematičnega modeliranja, povezane s študentskimi projekti. <p>V okviru vsebin so predstavljene tudi odprtokodne in komercialne tehnološke rešitve za obravnavo navedenih modelov. Za organizacijo, predstavitev in analizo podatkov se uporablja Excel. Linearni programi se rešujejo z reševalci v tehnologiji AMPL, Matlab, R ali Python, odvisno od okolja, iz katerega izhaja študentov problem.</p>	<p>The students are familiarized with open-source and commercial technological solutions for treatment of the studied mathematical models. Excel is used for initial data organization, presentation, and analysis, Students are introduced to different linear programming solvers: AMPL, Matlab, R or Python, depending on the environment the student's problem is coming from.</p>
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Temeljni literatura in viri / Readings:

Osnovno / basic:

- Osais, Yahya Esmail. *Computer Simulation: A Foundational Approach Using Python*. Chapman and Hall/CRC, 2017.
- R. Rardin. *Optimization in Operations Research*. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.
- J. Franklin, *Methods of Mathematical Economics: Linear and Nonlinear Programming, Fixed-Point Theorems*. Classics in Applied Mathematics 37, SIAM, 2002.
- Dossey, Giordano, McCrone, Weir, *Mathematics Methods and Modelling for today's Mathematics Classroom*, Brooks/Cole, Pacific Grove, 2002.

Dodatno / additional:

- E. Zakrajšek, *Matematično modeliranje*, DMFA – Založništvo, Ljubljana, 2004.
- J.D. Murray, *Mathematical biology I. An introduction*, Springer, New York, 2002.
- G. Polya, *Kako rešujemo matematične probleme*, DMFA, 1989.

Cilji in kompetence:

- Spoznati osnovne tehnike in prijeme matematičnega modeliranja po principih snovalskega razmišljanja.
- Spoznati teoretična ozadja matematičnega modeliranja in vpetost v druge znanstvene discipline.
- Uporabiti osnovne algoritme in heuristike za reševanje matematičnih problemov.
- Najti in uporabiti tehnološka orodja za izdelavo in raziskovanje matematičnega modela.
- Uporabiti znanje drugih matematičnih predmetov pri analizi praktičnih problemov.
- Pridobiti izkušnje pri izdelavi matematičnega modela, uporabnega pri kasnejši karieri.
- Pridobiti kompetenco iskanja in povzemanja literature o problemih, ki jih srečamo pri obravnavi modela.

Objectives and competences:

- Understanding of basic techniques of mathematical modeling using principles of design thinking.
- Acquaintance with the theoretical background of mathematical modeling.
- Find and apply technological tools to develop and investigate a mathematical model.
- Understanding of basic applications of algorithms and heuristics to solve mathematical problems.
- Apply the knowledge from other mathematical areas in analysis of practical problems.
- Gain experience in developing a mathematical model, useful in their future career.
- Learn about sources of bibliography on problems related to studying mathematical models.

- Seznaniti se z načini prepoznavanja za model pomembnih podatkov o problemu.
- Pridobiti izkušnje pri pojasnjevanju matematičnega modela in zagovarjanju njegovih predpostavk.

- Learn to distinguish the relevant data for the model under study.
- Gain experience in explaining and presenting the mathematical model and defending its assumptions.

Predvideni študijski rezultati:

Znanje in razumevanje:

- Poglobiti znanje iz matematičnih metod linearne optimizacije.
- Poglobiti znanje iz uporabe matematičnega modeliranja v znanosti in praksi.
- Podrobno poglobi znanje iz zahtevnejših aplikacij matematičnega modeliranja v finančni optimizaciji.

Prenesljive/ključne spretnosti in drugi atributi:

- Direktne aplikacije v finančni matematiki, ekonomiji, poslovnih vedah, inženirstvu, kemiji in številnih drugih družboslovnih in naravoslovnih vedah. Obenem principi linearne optimizacije tvorijo osnovo za matematično programiranje.
- Izdelava temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.

Intended learning outcomes:

Knowledge and Understanding:

- To deepen the knowledge of mathematical methods of linear optimization.
- To deepen the knowledge of applications of mathematical modelling in research and practice.
- To deepen the knowledge of details of advanced applications of mathematical modelling in financial optimization.

Transferable/Key Skills and other attributes:

- Direct applications in financial mathematics, economy, business, engineering, chemistry, and numerous other social and natural sciences. Also, principles of linear optimization are foundations for mathematical programming.
- Preparation of a detailed technical report or focused report paper describing a mathematical model of a specific mathematical problem the students encounter while investigation their possible future careers.

Metode poučevanja in učenja:

- Na predavanjih študentje spoznajo predpisano snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti.
- V okviru seminarskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja delajo na izbranih projektih.

Learning and teaching methods:

- At the lectures, the students are familiarized with the required contents of the course. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course.
- Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning on three smaller projects.

<ul style="list-style-type: none"> • V okviru seminarja študentje predstavijo rezultate projektov, s čimer se priučijo suverenega javnega nastopanja in zagovarjanja svojih rezultatov. 	<ul style="list-style-type: none"> • At the seminar, the students present the results of their projects, thus acquiring confidence with public presentation and defending their results. 	
Načini ocenjevanja:	Assessment:	
<p>Način (ustno izpraševanje, seminarska naloga - projekt):</p> <p>Seminarska naloga s tremi predstavitvami, ca. 90 ur samostojnega dela.</p> <p>Ustni izpit</p> <p>Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.</p> <p>Pozitivna ocena pri seminarski nalogi je pogoj za pristop k izpitu.</p>	<p>Delež (v %) / Weight (in %)</p> <p>75%</p> <p>25%</p>	<p>Type (oral examination, coursework - project):</p> <p>Coursework report, approx. 90 hours of individual work.</p> <p>Oral exam</p> <p>Each of the mentioned commitments must be assessed with a passing grade.</p> <p>Passing grade of the coursework report is required for taking the exam.</p>

Reference nosilca / Lecturer's references:

Glej COBISS/SICRIS. <http://sicris.izum.si/search/rsr.aspx?lang=slv&id=15413>

BOKAL, Drago, STEINBACHER, Mitja. Phases of psychologically optimal learning experience : task-based time allocation model. Central European Journal of Operations Research, ISSN 1435-246X, 2019, str. 1-23, doi: 10.1007/s10100-019-00609-0. [COBISS.SI-ID 24408328]

BOKAL, Drago, OPOROWSKI, Bogdan, RICHTER, Bruce, SALAZAR, Gelasio. Characterizing 2-crossing-critical graphs. Advances in applied mathematics, ISSN 0196-8858, 2016, vol. 74, str. 23-208. <http://dx.doi.org/10.1016/j.aam.2015.10.003>. [COBISS.SI-ID 17611353]

BOKAL, Drago, BREŠAR, Boštjan, JEREBIC, Janja. A generalization of Hungarian method and Hall's theorem with applications in wireless sensor networks. Discrete Applied Mathematics, ISSN 0166-218X. [Print ed.], 2012, vol. 160, iss. 4-5, str. 460-470. <http://dx.doi.org/10.1016/j.dam.2011.11.007>. [COBISS.SI-ID 16191577]

BOKAL, Drago, DEVOS, Matt, KLAVŽAR, Sandi, MIMOTO, Aki, MOOERS, Arne Ø. Computing quadratic entropy in evolutionary trees. Computers & Mathematics with Applications, ISSN 0898-1221. [Print ed.], 2011, vol. 62, no. 10, str. 3821-3828. <http://dx.doi.org/10.1016/j.camwa.2011.09.030>. [COBISS.SI-ID 16059481]

BOKAL, Drago, CHIMANI, Markus, LEANŐS, Jesús. Crossing number additivity over edge cuts. European journal of combinatorics, ISSN 0195-6698, 2013, vol. 34, iss. 6, str. 1010-1018. <http://dx.doi.org/10.1016/j.ejc.2013.02.002>. [COBISS.SI-ID 16624473]