



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

### UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Algebraična geometrija
<b>Course title:</b>	Algebraic geometry

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika, 3. stopnja		1. ali 2.	1. ali 2. ali 4.
Mathematics, 3 <sup>rd</sup> Degree		1 <sup>st</sup> or 2 <sup>nd</sup>	1 <sup>st</sup> or 2 <sup>nd</sup> or 4 <sup>th</sup>

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
60					240	10

**Nosilec predmeta / Lecturer:**

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	Slovenski / Slovene
	<b>Vaje / Tutorial:</b>	Slovenski / Slovene

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Poznanje osnovnih pojmov iz algebre in geometrije.

**Prerequisites:**

Knowledge of basic concepts from algebra and geometry.

**Vsebina:**

**Content (Syllabus outline):**

Algebraična geometrija je veda matematike, ki kombinira metode algebre, predvsem komutativne algebre, z jezikom in problemi geometrije. Veja sedi v centru moderne matematike in ima številne povezave s kompleksno analizo, topologijo in teorijo števil.

Predstavili bomo osnove komutativne algebre (lokalizacija, noetherski kolobarji, primarna dekompozicija, celostna zaprtja) in se nato osredotočili na algebraično geometrijo (ravninske stožnice, krivulje, afine varietete, Hilbertov Nullstellensatz, regularne funkcije na varietetah, projektivne varietete, biracionalna ekvivalenca, tangentni prostori, singularnost, dimenzija,...)

Navedena literatura služi le kot osnova in je nadgrajena z bolj specializiranimi teksti.

Algebraic geometry is a branch of mathematics which combines techniques of algebra, mainly commutative algebra, with the language and the problems of geometry. It occupies a central place in modern mathematics and has multiple connections with such diverse fields as complex analysis, topology and number theory.

We will present the basics of commutative algebra (localization, noetherian rings, primary decomposition, integral closure) and then focus on the algebraic geometry (planar conics, curves, affine varieties, Hilbert's Nullstellensatz, regular functions on varieties, projective varieties, birational equivalence, tangent spaces, singularity, dimension ,...)

The literature cited generally serves as a base and is being upgraded with more specialized texts.

### Temeljni literatura in viri / Readings:

- Michael Atiyah & Ian G. MacDonald (1969), Introduction to Commutative Algebra, Addison-Wesley Publishing
- David Eisenbud (1999), Commutative Algebra With a View Toward Algebraic Geometry, Springer-Verlag,
- Miles Reid (1988). Undergraduate Algebraic Geometry. Cambridge University Press.
- Robin Hartshorne (1997). Algebraic Geometry. Springer-Verlag.
- Oscar Zariski, Pierre Samuel, Pierre (1960) Commutative algebra. Vol. 1, 2., Springer-Verlag.

### Cilji in kompetence:

- Doseči poglobljeno razumevanje teoretskih in metodoloških konceptov s področja komutativne algebre in algebraične geometrije
- Razviti sposobnost samostojnega razvijanja novega znanja s področja komutativne algebre in algebraične geometrije
- Razviti sposobnost za samostojno reševanje najzahtevnejših problemov iz komutativne algebre in algebraične geometrije
- Razviti sposobnost izboljševanja znanih in odkrivanja novih rezultatov s področja komutativne algebre in algebraične geometrije
- Zmožnost razvijanja kritične refleksije na področju komutativne algebre in algebraične geometrije
- Razviti zmožnost vodenja najzahtevnejših znanstvenoraziskovalnih projektov s širšega področja komutativne algebre in algebraične geometrije.

### Objectives and competences:

- To achieve a deeper understanding of theoretical and methodological concepts of commutative algebra and algebraic geometry
- To develop the ability to independently develop new knowledge in the field of commutative algebra and algebraic geometry
- To develop the ability for solving the most challenging problems in commutative algebra and algebraic geometry
- To develop the ability of improving known results as well as obtaining new results in commutative algebra and algebraic geometry
- Ability to develop critical reflection in commutative algebra and algebraic geometry
- To develop the ability to lead the most challenging scientific research projects in the wider field of commutative algebra and algebraic geometry

### Predvideni študijski rezultati:

### Intended learning outcomes:

**Znanje in razumevanje:**

- poznavanje in razumevanje osnovnih rezultatov komutativne algebre in algebraične geometrije;
- poznavanje algoritmičnih prijemov iz algebre in njihova implementacija..

**Prenesljive/ključne spretnosti in drugi atributi:**

- podlaga za raziskovalno delo na področju algebre in algebraične geometrije;
- prenos in implementacija znanja iz algebre različna strokovna in znanstvena področja, kjer se uporabljajo algebraične metode.

**Knowledge and understanding:**

- knowledge and understanding of basic results of commutative algebra and algebraic geometry;
- knowledge and understanding of basic algorithmic approaches to algebra and their implementations.

**Transferable/Key Skills and other attributes:**

- a basis for research in area of algebra and algebraic geometry;
- implementation and knowledge transfer of statistical methods into different areas dealing with algebraic methods.

**Metode poučevanja in učenja:**

- predavanja;
- reševanje praktičnih nalog;
- priprava seminarja;
- konzultacije;
- samostojni študij.

**Learning and teaching methods:**

- lectures;
- solving concrete problems;
- seminar work;
- consultations;
- self-study.

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- seminarsko predavanje;
- rešitve praktičnih nalog;
- ustni ali pisni izpit.

30

30

40

Type (written examination, oral exam, coursework, project):

- seminar talk;
- solutions of concrete problems;
- **oral or written examination.**

**Reference nosilca / Lecturer's references:**

The Procesi-Schacher conjecture and Hilbert's 17th problem for algebras with involution, J. Algebra, 2010, vol. 324, no. 2, pp. 256-268. (with Thomas Unger).

Free convex algebraic geometry, "Semidefinite Optimization and Convex Algebraic Geometry" edited by G. Blekherman, P. Parrilo, R. Thomas, pp. 341-405, SIAM, 2013. (with J. William Helton and Scott McCullough).

A local-global principle for linear dependence of noncommutative polynomials, Israel J. Math., 2013, vol. 193, pp. 71-82. (with Matej Brešar).

On real one-sided ideals in a free algebra, J. Pure Appl. Algebra, 2014, vol. 218, pp. 269-284. (with Jaka Cimpric, J. William Helton, Scott McCullough and Chris Nelson).

An exact duality theory for semidefinite programming based on sums of squares, Math. Oper. Res., 2013, vol. 38, pp. 569-590. (with Markus Schweighofer).