



Univerza v Mariboru

Fakulteta za naravoslovje
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS

Predmet:	Realna algebraična geometrija
Course title:	Real algebraic geometry

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika, 3. stopnja		1. ali 2.	1. ali 2. ali 4.
Mathematics, 3 rd Degree		1 st or 2 nd	1 st or 2 nd or 4 th

Vrsta predmeta / Course type

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
60					240	10

Nosilec predmeta / Lecturer:

Jeziki / Languages:	Predavanja / Lectures:	Slovenski / Slovene
	Vaje / Tutorial:	Slovenski / Slovene

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Poznanje osnovnih pojmov iz algebre.

Prerequisites:

Knowledge of basic concepts from algebra.

Vsebina:

Content (Syllabus outline):

Realna algebraična geometrija preučuje semialgebraične množice, ki so rešitve končnih sistemov polinomskih enačb in neenačb (v več spremenljivkah, z realnimi koeficienti). Obravnavali bomo naslednje teme:

- urejeni obsegi, realno zaprti obsegi;
- *princip Tarskega*, ki pravi, da je projekcija semialgebraične množice vedno semialgebraična množica;
- strukturni izrek za semialgebraične množice, ki mu pravimo *Cilindrična algebraična dekompozicija*. Posledica tega izreka je, da so semialgebraične množice triangularizabilne;
- algebraične karakterizacije polinomov, ki so pozitivni oziroma nenegativni na dani bazični semialgebraični množici ter njihovo uporabo v optimizaciji polinomskih funkcij.

Navedena literatura služi le kot osnova in je nadgrajena z bolj specializiranimi teksti.

Real algebraic geometry studies semialgebraic sets which are solution sets of polynomial equations and inequalities (in several variables with real coefficients). We will address the following topics:

- ordered fields, real closed fields;
- the Tarski principle, which says that a projection of a semialgebraic set is itself semialgebraic. We will also discuss important implications of this result;
- cylindrical algebraic decomposition which is a classification result for semialgebraic sets. In particular, it implies that the latter sets are triangularizable;
- algebraic characterization of polynomials that are positive or nonnegative or zero on a given semialgebraic set, and their use in the optimization of polynomial functions.

The literature cited generally serves as a base and is being upgraded with more specialized texts.

Temeljni literatura in viri / Readings:

- C. Andradas, L. Bröcker, J.-M. Ruiz, *Constructible sets in real geometry*, *Ergebnisse der Mathematik und ihrer Grenzgebiete (3)*, 33. Springer-Verlag, Berlin, 1996.
- J. Bochnak, M. Coste, M.-F. Roy, *Real algebraic geometry*, *Ergebnisse der Mathematik und ihrer Grenzgebiete (3)* 36. Springer-Verlag, Berlin, 1998.
- M. Coste, *Introduction to semialgebraic geometry*, Dip. Mat. Univ. Pisa, Dottorato di Ricerca in Matematica, Istituti Editoriali e Poligrafici Internazionali, Pisa (2000), <http://perso.univ-rennes1.fr/michel.coste/polyens/SAG.pdf>.
- A. Prestel, C.-N. Delzell, *Positive polynomials*. From Hilbert's 17th problem to real algebra, Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2001.
- C. Scheiderer, *Real algebraic geometry*, skripta, <http://www.math.uni-konstanz.de/~scheider/vorles/0708ws/RAG.html>

Cilji in kompetence:

- Doseči poglobljeno razumevanje teoretskih in metodoloških konceptov s področja realne algebraične geometrije
- Razviti sposobnost samostojnega razvijanja novega znanja s področja realne algebraične geometrije
- Razviti sposobnost za samostojno reševanje najzahtevnejših problemov iz realne algebraične geometrije
- Razviti sposobnost izboljševanja znanih in odkrivanja novih rezultatov s področja realne algebraične geometrije
- Zmožnost razvijanja kritične refleksije na področju realne algebraične geometrije
- Razviti zmožnost vodenja najzahtevnejših znanstvenoraziskovalnih projektov s širšega področja realne algebraične geometrije.

Objectives and competences:

- To achieve a deeper understanding of theoretical and methodological concepts of real algebraic geometry
- To develop the ability to independently develop new knowledge in the field of real algebraic geometry
- To develop the ability for solving the most challenging problems in real algebraic geometry
- To develop the ability of improving known results as well as obtaining new results in real algebraic geometry
- Ability to develop critical reflection in real algebraic geometry
- To develop the ability to lead the most challenging scientific research projects in the wider field of real algebraic geometry

Predvideni študijski rezultati:Znanje in razumevanje:

- poznavanje in razumevanje osnovnih rezultatov realne algebre in realne algebraične geometrije;
- poznavanje algoritmičnih prijemov iz algebre in njihova implementacija..

Prenesljive/ključne spretnosti in drugi atributi:

- podlaga za raziskovalno delo na področju algebre;
- prenos in implementacija znanja iz algebre različna strokovna in znanstvena področja, kjer se uporabljajo algebraične metode.

Intended learning outcomes:Knowledge and understanding:

- knowledge and understanding of basic results of real algebra and real algebraic geometry;
- knowledge and understanding of basic algorithmic approaches to algebra and their implementations.

Transferable/Key Skills and other attributes:

- a basis for research in area of algebra;
- implementation and knowledge transfer of statistical methods into different areas dealing with algebraic methods.

Metode poučevanja in učenja:

- predavanja;
- reševanje praktičnih nalog;
- priprava seminarja;
- konzultacije;
- samostojni študij.

Learning and teaching methods:

- lectures;
- solving concrete problems;
- seminar work;
- consultations;
- self-study.

Delež (v %) /

Weight (in %)

Načini ocenjevanja:**Assessment:**Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- seminarsko predavanje;
- rešitve praktičnih nalog;
- ustni ali pisni izpit.

30**30****40**Type (written examination, oral exam, coursework, project):

- seminar talk;
- solutions of concrete problems;
- **oral or written examination.**

Reference nosilca / Lecturer's references:

The Procesi-Schacher conjecture and Hilbert's 17th problem for algebras with involution, J. Algebra, 2010, vol. 324, no. 2, pp. 256-268. (with Thomas Unger).

The convex Positivstellensatz in a free algebra, Adv. Math., 2012, vol. 231, pp. 516-534. (with J. William Helton and Scott McCullough).

A local-global principle for linear dependence of noncommutative polynomials, Israel J. Math., 2013, vol. 193, pp. 71-82. (with Matej Brešar).

On real one-sided ideals in a free algebra, J. Pure Appl. Algebra, 2014, vol. 218, pp. 269-284. (with Jaka Cimpric, J. William Helton, Scott McCullough and Chris Nelson).

An exact duality theory for semidefinite programming based on sums of squares, Math. Oper. Res., 2013, vol. 38, pp. 569-590. (with Markus Schweighofer).