Univerza v Mariboru

Fakulteta za naravoslovje in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	Teorija mere
Course title:	Measure theory

Študijski program in stopnja Study programme and level	**Študijska smer Study field**	**Letnik Academic year**	**Semester Semester**
Matematika, 2. stopnja	Modul S1	**1. ali 2.**	**1. ali 3.**
Mathematics, 2nd degree	Module S1	**1. or 2.**	**1. or 3.**

Vrsta predmeta / Course type

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	**Seminar Seminar**	**Sem. vaje Tutorial**	**Lab. vaje Laboratory work**	**Teren. vaje Field work**	**Samost. delo Individ. work**	**ECTS**
60		**45**			**165**	**9**

Nosilec predmeta / Lecturer: Valerij Romanovskij

Jeziki / Languages:	**Predavanja / Lectures:**	SLOVENSKO/SLOVENE
	Vaje / Tutorial:	SLOVENSKO/SLOVENE

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	**Prerequisits:**

Vsebina:	**Content (Syllabus outline):**
• Osnovni pojmi teorije mere: Algebra, σ-algebra, Borelova σ-algebra na R^n. Mere in osnovne lastnosti mer. Merljivi prostori. Pozitivne mere. Zunanje mere. Lebesqueova mera na R^n. • Funkcije in integrali: Merljive funkcije. Stopničaste funkcije. Integral stopničaste funkcije. Integral merljive funkcije. Izrek o monotoni konvergenci. Fatoujeva lema in Lebesqueov izrek o dominantni konvergenci. Povezanost Riemannovega in Lebesqueovega integrala.	• Basic concepts of measure theory: Algebra, σ-algebra, Borel σ-algebra on R^n. Measure and its basic properties. Measurable spaces. Positive measures. Outer measures. Lebesque measure on R^n. • Functions and integrals: Measurable functions. Simple measurable functions. The integral of a simple measurable function. The integral of a measurable function. The monotone convergence theorem. Fatou's lemma and Lebesque's dominated convergence theorem. Relationships between

- Konvergenca: Zaporedja merljivih funkcij in konvergenca. Konvergenca skoraj povsod. Norma in normirani L^p-prostori. Neenakosti (Hölder, Minkowski). Dualni prostori.
- Predznačne in kompleksne mere: Predznačne mere in Hahnov razcepni izrek. Kompleksne mere in Radon-Nikodymov izrek. Funkcije z omejeno varianco.
- Produktne mere: Merjenje in integriranje po produktnih prostorih (Fubinijev izrek).
- Odvajanje: Odvodi mer. Odvodi funkcij.
- Rieszov izrek o reprezentaciji pozitivnih linearnih funkcionalov na C(X).
- *Lebesgue-Stieltjesov integral.*

Riemann's and Lebesque's integral.

- Convergence: Sequences of measurable functions and convergence. Convergence almost everywhere. Norm and normed L^p-spaces. Inequalities (Hölder, Minkowski). Dual spaces.
- Signed and complex measures: Signed measures and the Hahn decomposition theorem. Complex measures and the Radon-Nikodym theorem. Functions of bounded variation.
- Product measures: Measures and integrals on product spaces (Fubini's theorem).
- Differentiation: Differentiation of measures. Differentiation of functions.
- The Riesz representation theorem on positive linear functionals on C(X).

Lebesgue-Stieltjes integral

Temeljni literatura in viri / Readings:

1. M. Capinski, E. Kopp: *Measure, integral and probability*, Springer-Verlag London, 2004.
2. D. L. Cohn: *Measure theory*, Birkhäuser, 1994.
3. R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA, 2001.
4. M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA, 1985.
5. W. Rudin: *Real and complex analysis, 3th edition*, Mc-Graw-Hill, 1986.
6. H. Sohrab, *Basic real analysis*, Birkhauser Boston, 2003.
7. I. Vidav, *Višja matematika II*, DZS, Ljubljana, 1975.

Cilji in kompetence:

Glavni cilj predmeta je proučiti temeljne koncepte in rezultate teorije mere.

Objectives and competences:

The main goal of the course is to study the fundamental concepts and results of measure theory.

Predvideni študijski rezultati:

Znanje in razumevanje:

- merljivi prostori, merljive funkcije, abstraktno integriranje, izreki o konvergenci, L^p-prostori, produktne mere, odvodi mer.

Prenesljive/ključne spretnosti in drugi atributi:

- Poznavanje osnov teorije mere je podlaga za študij različnih matematičnih področij (funkcionalne analize, verjetnosti, parcialnih diferencialnih enačb itd.).

Intended learning outcomes:

Knowledge and Understanding:

- Measurable spaces, measurable functions, abstract integration, convergence theorems, L^p-spaces, product measures, differentiation of measures.

Transferable/Key Skills and other attributes:

- Knowing the fundamentals of measure theory is a prerequisite for studying various mathematical areas (functional analysis, probability, partial differential equations etc.).

Metode poučevanja in učenja:	Learning and teaching methods:
• Predavanja • Teoretične vaje	• Lectures • Theoretical exercises

Načini ocenjevanja:		Assessment:
Način (pisni izpit, ustno izpraševanje, naloge, projekt) • Pisni izpit – problemi • Pisni izpit – teoretija Pisni izpit - problemi se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) Pisni izpit - teorja se lahko nadomesti z dvema delnima testoma (sprotni obveznosti) Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.	Delež (v %) / Weight (in %) 50% 50%	Type (examination, oral, coursework, project): • Written exam – problems • Written exam – theory Written exam – problems can be replaced by two parital tests (mid-term testing) Written exam – theory can be replaced by two parital tests (mid-term testing) Each of the mentioned commitments must be assessed with a passing grade.
Reference nosilca / Lecturer's references:		

1. CHEN, Xingwu, GINÉ, Jaume, ROMANOVSKI, Valery, SHAFER, Douglas. The 1: -q resonant center problem for certain cubic Lotka-Volterra systems. *Appl. math. comput.*. [Print ed.], Aug. 2012, vol. 218, iss. 32, str. 11620-11633. http://dx.doi.org/10.1016/j.amc.2012.05.045, doi: 10.1016/j.amc.2012.05.045. [COBISS.SI-ID 19321352]

2. BASOV, Vladimir V., ROMANOVSKI, Valery. Linearization of two-dimensional systems of ODEs without conditions on small denominators. *Appl. math. lett.*. [Print ed.], 2012, vol. 25, iss. 2, str. 99-103. http://dx.doi.org/10.1016/j.aml.2011.06.029, doi: 10.1016/j.aml.2011.06.029. [COBISS.SI-ID 18675208]

3. LEVANDOVSKYY, Viktor, PFISTER, Gerhard, ROMANOVSKI, Valery. Evaluating cyclicity of cubic systems with algorithms of computational algebra. *Commun. pure appl. anal.*, 2012, vol. 11, no. 5, str. 2023-2035, doi: 10.3934/cpaa.2012.11.2023. [COBISS.SI-ID 19075080]

4. WENTAO, Huang, CHEN, Xingwu, ROMANOVSKI, Valery. Linear centers with perturbations of degree 2d + 5. *Int. j. bifurc. chaos appl. sci. eng.*, 2012, vol. 22, no. 1, str. [1250007-1 - 1250007-12]. http://www.ejournals.wspc.com.sg/ijbc/22/2201/S0218127412500071.html, doi: 10.1142/S0218127412500071. [COBISS.SI-ID 69213185]

5. HAN, Maoan, ROMANOVSKI, Valery. Isochronicity and normal forms of polynomial systems of ODEs. *J. symb. comput.*, Oct. 2012, vol. 47, iss. 10, str. 1163-1174. http://dx.doi.org/10.1016/j.jsc.2011.12.039, doi: 10.1016/j.jsc.2011.12.039. [COBISS.SI-ID 19324168]