



Univerza v Mariboru

Fakulteta za naravoslovje
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS

Predmet:	Teorija mere
Course title:	Measure theory

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika, 2. stopnja	Modul F1	1. ali 2.	1. ali 3.
Mathematics, 2 nd degree	Module F1	1. or 2.	1. or 3.

Vrsta predmeta / Course type

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
60		45			165	9

Nosilec predmeta / Lecturer:

Jeziki / Languages:	Predavanja / Lectures:	SLOVENSKO/SLOVENE
	Vaje / Tutorial:	SLOVENSKO/SLOVENE

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Prerequisites:

Vsebina:

- Osnovni pojmi teorije mere: Algebra, σ -algebra, Borelova σ -algebra na \mathbb{R}^n . Mere in osnovne lastnosti mer. Merljivi prostori. Pozitivne mere. Zunanje mere. Lebesgueova mera na \mathbb{R}^n .
- Funkcije in integrali: Merljive funkcije. Stopničaste funkcije. Integral stopničaste funkcije. Integral merljive funkcije. Izrek o monotoni konvergenci. Fatoujeva lema in Lebesgueov izrek o dominantni konvergenci. Povezanost Riemannovega in Lebesgueovega integrala.

Content (Syllabus outline):

- Basic concepts of measure theory: Algebra, σ -algebra, Borel σ -algebra on \mathbb{R}^n . Measure and its basic properties. Measurable spaces. Positive measures. Outer measures. Lebesgue measure on \mathbb{R}^n .
- Functions and integrals: Measurable functions. Simple measurable functions. The integral of a simple measurable function. The integral of a measurable function. The monotone convergence theorem. Fatou's lemma and Lebesgue's dominated convergence theorem. Relationships between

- Konvergenca: Zaporedja merljivih funkcij in konvergenca. Konvergenca skoraj povsod. Norma in normirani L^p -prostori. Neenakosti (Hölder, Minkowski). Dualni prostori.
- Predznačne in kompleksne mere: Predznačne mere in Hahnov razcepni izrek. Kompleksne mere in Radon-Nikodymov izrek. Funkcije z omejeno varianco.
- Produktne mere: Merjenje in integriranje po produktnih prostorih (Fubinijev izrek).
- Odvajanje: Odvodi mer. Odvodi funkcij.
- Rieszov izrek o reprezentaciji pozitivnih linearnih funkcionalov na $C(X)$.
- *Lebesgue-Stieltjesov integral*.

- Riemann's and Lebesgue's integral.
- Convergence: Sequences of measurable functions and convergence. Convergence almost everywhere. Norm and normed L^p -spaces. Inequalities (Hölder, Minkowski). Dual spaces.
 - Signed and complex measures: Signed measures and the Hahn decomposition theorem. Complex measures and the Radon-Nikodym theorem. Functions of bounded variation.
 - Product measures: Measures and integrals on product spaces (Fubini's theorem).
 - Differentiation: Differentiation of measures. Differentiation of functions.
 - The Riesz representation theorem on positive linear functionals on $C(X)$. *Lebesgue-Stieltjes integral*

Temeljni literatura in viri / Readings:

1. M. Capinski, E. Kopp: *Measure, integral and probability*, Springer-Verlag London, 2004.
2. D. L. Cohn: *Measure theory*, Birkhäuser, 1994.
3. R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA, 2001.
4. M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA, 1985.
5. W. Rudin: *Real and complex analysis, 3th edition*, Mc-Graw-Hill, 1986.
6. H. Sohrab, *Basic real analysis*, Birkhauser Boston, 2003.
7. I. Vidav, *Višja matematika II*, DZS, Ljubljana, 1975.

Cilji in kompetence:

Glavni cilj predmeta je proučiti temeljne koncepte in rezultate teorije mere.

Objectives and competences:

The main goal of the course is to study the fundamental concepts and results of measure theory.

Predvideni študijski rezultati:

Znanje in razumevanje:

- merljivi prostori, merljive funkcije, abstraktno integriranje, izreki o konvergenci, L^p -prostori, produktne mere, odvodi mer.

Prenesljive/ključne spretnosti in drugi atributi:

- Poznavanje osnov teorije mere je podlaga za študij različnih matematičnih področij (funkcionalne analize, verjetnosti, parcialnih diferencialnih enačb itd.).

Intended learning outcomes:

Knowledge and Understanding:

- Measurable spaces, measurable functions, abstract integration, convergence theorems, L^p -spaces, product measures, differentiation of measures.

Transferable/Key Skills and other attributes:

- Knowing the fundamentals of measure theory is a prerequisite for studying various mathematical areas (functional analysis, probability, partial differential equations etc.).

Metode poučevanja in učenja:		Learning and teaching methods:	
<ul style="list-style-type: none"> • Predavanja • Teoretične vaje 		<ul style="list-style-type: none"> • Lectures • Theoretical exercises 	
Načini ocenjevanja:		Assessment:	
<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt)</p> <ul style="list-style-type: none"> • Pisni izpit – problemi • Pisni izpit – teoretija <p>Pisni izpit - problemi se lahko nadomesti z dvema delnima testoma (sprotni obveznosti)</p> <p>Pisni izpit - teorija se lahko nadomesti z dvema delnima testoma (sprotni obveznosti)</p> <p>Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.</p>	<p>Delež (v %) / Weight (in %)</p> <p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> • Written exam – problems • Written exam – theory <p>Written exam – problems can be replaced by two parital tests (mid-term testing)</p> <p>Written exam – theory can be replaced by two parital tests (mid-term testing)</p> <p>Each of the mentioned commitments must be assessed with a passing grade.</p>	
Reference nosilca / Lecturer's references:			
<p>1. CHEN, Xingwu, GINÉ, Jaume, ROMANOVSKI, Valery, SHAFER, Douglas. The 1: -q resonant center problem for certain cubic Lotka-Volterra systems. <i>Appl. math. comput.</i> [Print ed.], Aug. 2012, vol. 218, iss. 32, str. 11620-11633. http://dx.doi.org/10.1016/j.amc.2012.05.045, doi: 10.1016/j.amc.2012.05.045. [COBISS.SI-ID 19321352]</p> <p>2. BASOV, Vladimir V., ROMANOVSKI, Valery. Linearization of two-dimensional systems of ODEs without conditions on small denominators. <i>Appl. math. lett.</i> [Print ed.], 2012, vol. 25, iss. 2, str. 99-103. http://dx.doi.org/10.1016/j.aml.2011.06.029, doi: 10.1016/j.aml.2011.06.029. [COBISS.SI-ID 18675208]</p> <p>3. LEVANDOVSKYY, Viktor, PFISTER, Gerhard, ROMANOVSKI, Valery. Evaluating cyclicity of cubic systems with algorithms of computational algebra. <i>Commun. pure appl. anal.</i>, 2012, vol. 11, no. 5, str. 2023-2035, doi: 10.3934/cpaa.2012.11.2023. [COBISS.SI-ID 19075080]</p> <p>4. WENTAO, Huang, CHEN, Xingwu, ROMANOVSKI, Valery. Linear centers with perturbations of degree $2d + 5$. <i>Int. j. bifurc. chaos appl. sci. eng.</i>, 2012, vol. 22, no. 1, str. [1250007-1 - 1250007-12]. http://www.ejournals.wspc.com.sg/ijbc/22/2201/S0218127412500071.html, doi: 10.1142/S0218127412500071. [COBISS.SI-ID 69213185]</p> <p>5. HAN, Maoan, ROMANOVSKI, Valery. Isochronicity and normal forms of polynomial systems of ODEs. <i>J. symb. comput.</i>, Oct. 2012, vol. 47, iss. 10, str. 1163-1174. http://dx.doi.org/10.1016/j.jsc.2011.12.039, doi: 10.1016/j.jsc.2011.12.039. [COBISS.SI-ID 19324168]</p>			