



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

### UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Matematično modeliranje
<b>Course title:</b>	Mathematical modelling

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika	Uporabna matematika	2	4
Mathematics	Applied mathematics	2	4

Vrsta predmeta / Course type:

Univerzitetna koda predmeta / University course code:

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15		15		120	6

Nosilec predmeta / Lecturer:

Jeziki / Languages:	Predavanja / Lectures:	SLOVENSKO/SLOVENE ali Angleško/English
	Vaje / Tutorial:	SLOVENSKO/SLOVENE ali Angleško/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Poznavanje enostavnih algoritmov.  
Poznavanje osnov linearne algebre in vektorske analize.

**Prerequisites:**

Knowledge of simple algorithms.  
Knowledge of basic linear algebra and calculus.

**Vsebina:**

**Content (Syllabus outline):**

Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav:

- Pregled vrst matematičnih modelov. Proces izdelave matematičnega modela. Vrste spremenljivk.
- Matematično modeliranje in inovacijski proces.
- Modeliranje odločitev, odločitveno drevo, razveji in omeji, dinamično programiranje.
- Optimizacijski modeli s centraliziranim odločanjem, modeli teorije iger s porazdeljenim odločanjem. Deterministični, stohastični, robustni problemi.
- Linearni program. Dual. Simpleksna metoda.
- Farkaseva lema. Senčne cene. Analiza občutljivosti.
- Stohastični linearni program (diskretna spremenljivka). Dekompozicija.
- Uvod v teorijo iger. Nashevo ravnovesje. Matrične igre. Igre z ničelno vsoto.
- Simulacijski modeli. Modeliranje sprememb z diferenčnimi in diferencialnimi enačbami.
- Matematično obnašanje dinamičnih sistemov.
- Analiza podatkov, verjetnost, Monte Carlo simulacija.

V okviru obvezne vsebine študentje izdelajo tri krajše seminarske naloge, preko katerih utrdijo poznavanje procesa matematičnega modeliranja. Naloge so povezane z njihovo bodočo kariero (praktični problemi iz gospodarstva, teoretični problemi iz teorije modeliranja, optimizacije, algoritmov). Preostala predavanja se prilagodijo projektu, ki so jih izbrali študentje, in obsegajo naslednje vsebine:

- Deterministični in stohastični modeli optimizacije portfelja.
- Problem prehrane.
- Aplikacije teorije iger: optimalna strategija na tržišču z dvema konkurentoma.
- Čakalne vrste.
- Druge vsebine s področja matematičnega modeliranja, povezane s študentskimi projekti.

V okviru vsebin so predstavljene tudi odprtokodne in komercialne tehnološke rešitve za obravnavo navedenih modelov.

Mandatory content that familiarizes the students with fundamentals of mathematical modeling:

- Overview of mathematical model types. Process of mathematical model creation. Variable types.
- Mathematical model and innovation process.
- Decision modeling. Decision tree. Branch and bound. Dynamic programming.
- Optimization models with centralized decision making. Game theory models with distributed decision making. Deterministic, stochastic, robust problems.
- Linear program and its dual. Simplex method.
- Farkash lemma. Shadow prices. Sensitivity analysis.
- Stochastic linear program (discrete variable). Decomposition.
- Introduction to game theory. Nash equilibria. Matrix zero sum games.
- Simulation models. Modeling changes with difference and differential equations.
- Mathematical behaviour of dynamic systems.
- Data analysis, probability, monte carlo simulations.

Within the coursework, the students select smaller problems whose result are coursework reports. The problems are related to their future career (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list:

- Deterministic and stochastic models of portfolio optimization.
- Diet problem.
- Applications of game theory: optimal strategy in two competitor market.
- Queues.
- Other material from the field of mathematical modeling, related to students' projects.

The students are familiarized with open-source and commercial technological solutions for treatment of the studied mathematical models.

### Temeljni literatura in viri / Readings:

#### Osnovno / basic:

- R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.
- J. Franklin, Methods of Mathematical Economics: Linear and Nonlinear Programming, Fixed-Point Theorems. Classics in Applied Mathematics 37, SIAM, 2002.
- Dossey, Giordano, McCrone, Weir, Mathematics Methods and Modelling for today's Mathematics Classroom, Brooks/Cole, Pacific Grove, 2002.

#### Dodatno / additional:

- E. Zakrajšek, *Matematično modeliranje*, DMFA – Založništvo, Ljubljana, 2004.

- J.D. Murray, *Mathematical biology I. An introduction*, Springer, New York, 2002.
- G. Polya, *Kako rešujemo matematične probleme*, DMFA, 1989.

### Cilji in kompetence:

Usvojiti proces matematičnega modeliranja.

Razviti kompetenco samostojnega apliciranja matematičnih metod na probleme iz finančne optimizacije, ekonomije, ter širše iz gospodarstva.

Spoznati tehnološka orodja, s katerimi se srečujemo pri reševanju optimizacijskih problemov in problemov matematičnega modeliranja.

### Objectives and competences:

Familiarize the students with the process of mathematical modelling.

Develop competent skills of independent application of mathematical methods to the problems from financial optimization, economics, and broader from industry.

Familiarize the students with technological tools that assist solving optimization problems and problems related to mathematical modelling.

### Predvideni študijski rezultati:

Znanje in razumevanje:

- Poglobiti znanje iz matematičnih metod linearne deterministične in stohastične optimizacije.
- Poglobiti znanje iz uporabe matematičnega modeliranja v znanosti in praksi.
- Podrobno poglobi znanje iz zahtevnejših aplikacij operacijskih raziskav v finančni optimizaciji.

Prenesljive/ključne spretnosti in drugi atributi:

- Direktno aplikacije v finančni matematiki, ekonomiji, poslovnih vedah, inženirstvu, kemiji in številnih drugih družboslovnih in naravoslovnih vedah. Obenem principi linearne optimizacije tvorijo osnovo za matematično programiranje.

### Intended learning outcomes:

Knowledge and Understanding:

- To deepen the knowledge of mathematical methods of linear deterministic and stochastic optimization.
- To deepen the knowledge of applications of operations research to economics and numerous other fields.
- To deepen the knowledge of details of advanced applications of operations research in financial optimization.

Transferable/Key Skills and other attributes:

- Direct applications in financial mathematics, economy, business, engineering, chemistry, and numerous other social and natural sciences. Also, principles of linear optimization are foundations for mathematical optimization.

### Metode poučevanja in učenja:

- Na predavanjih študentje spoznajo predpisano snov predmeta.
- V okviru seminarjskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja delajo na izbranih projektih.
- V okviru seminarja študentje predstavijo rezultate projektov, s čimer se priučijo suverenega javnega nastopanja in zagovarjanja svojih rezultatov.

### Learning and teaching methods:

- At the lectures the students are familiarized with the required contents of the course.
- Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning on three smaller projects.
- At the seminar, the students present the results of their projects, thus acquiring confidence with public presentation and defending their results.

**Načini ocenjevanja:****Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt) Tri seminarske naloge, ca. 30 ur samostojnega dela z vsako.	Delež (v %) / Weight (in %) 25%, 25%, 25%	Type (examination, oral, coursework, project): Three coursework reports, approx. 30 hours of individual work each
Ustni izpit	25%	Oral exam
Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.		Each of the mentioned commitments must be assessed with a passing grade.
Pozitivna ocena pri seminarskih nalogah je pogoj za pristop k izpitu.		Passing grade of the coursework reports is required for taking the exam.

**Reference nosilca / Lecturer's references:**

Glej COBISS/SICRIS.

<http://sicris.izum.si/search/rsr.aspx?lang=slv&id=15413>

BOKAL, Drago, BREŠAR, Boštjan, JEREBIC, Janja. A generalization of Hungarian method and Hall's theorem with applications in wireless sensor networks. *Discrete Applied Mathematics*, ISSN 0166-218X. [Print ed.], 2012, vol. 160, iss. 4-5, str. 460-470.

<http://dx.doi.org/10.1016/j.dam.2011.11.007>. [COBISS.SI-ID 16191577]

BOKAL, Drago, DEVOS, Matt, KLAVŽAR, Sandi, MIMOTO, Aki, MOOERS, Arne Ø. Computing quadratic entropy in evolutionary trees. *Computers & Mathematics with Applications*, ISSN 0898-1221. [Print ed.], 2011, vol. 62, no. 10, str. 3821-3828.

<http://dx.doi.org/10.1016/j.camwa.2011.09.030>. [COBISS.SI-ID 16059481]

ŽUNKO, Matjaž, BOKAL, Drago, JAGRIČ, Timotej. Testiranje modelov VaR v izjemnih okoliščinah. *IB revija*, ISSN 1318-2803. [Slovenska izd.], 2011, letn. 45, št. 3, str. 57-67, tabele, graf. prikazi. [COBISS.SI-ID 10777884]

BOKAL, Drago, CZABARKA, Éva, SZÉKELY, László, VRT'Ó, Imrich. General lower bounds for the minor crossing number of graphs. *Discrete & computational geometry*, ISSN 0179-5376, 2010, vol. 44, no. 2, str. 463-483. <http://dx.doi.org/10.1007/s00454-010-9245-4>. [COBISS.SI-ID 15636057]

BOKAL, Drago, CHIMANI, Markus, LEANŐS, Jesús. Crossing number additivity over edge cuts. *European journal of combinatorics*, ISSN 0195-6698, 2013, vol. 34, iss. 6, str. 1010-1018. <http://dx.doi.org/10.1016/j.ejc.2013.02.002>. [COBISS.SI-ID 16624473]