Phantom mappings and a shape-theoretic problem concerning products

In 1977 Y. Kodama proved that the Cartesian product of an FANR (compact metric space shape dominated by a compact polyhedron) and a paracompact space is a product in the shape category of topological spaces Sh(Top). In the same paper he stated the following problem, which is still open.

PROBLEM 1. Let X be a movable continuum and let P be a metric space. Is the Cartesian product $X \times P$ a product in Sh(Top)?

Since the simplest movable continuum, which is not an FANR, is the Hawaiian earring, it is natural to try to solve the following (also open) problem.

PROBLEM 2. Let X be the Hawaiian earring and let P be the wedge of a sequence of copies of the 1-sphere $S * 1$ (weak topology). Is $X \times P$ a product in Sh(Top)?

The next (open) problem is a simplification of Problem 2. A positive answer would imply a negative answer to problems 2 and 1.

PROBLEM 3. Let X an P be as in Problem 2 and let Z be a polyhedron. Is there a nontrivial shape morphism $H\colon Z \to X \times P$, whose compositions with the canonical projections $\pi_X\colon X \times P \to X$ and $\pi_P\colon X \times P \to P$ are constant shape morphisms $*\colon Z \to X$ and $*\colon Z \to X$.

We construct a polyhedral resolution $\boldsymbol{q}\colon X \times P \to \boldsymbol{Y} = (Y_\mu)$ and show that Problem 3 is equivalent to the following Problem 4, involving phantom mappings (restrictions to compact subsets are homotopic to constant mappings $*$).

PROBLEM 4. Is there a nontrivial homotopy mapping $\boldsymbol{h} = (h_\mu)\colon \boldsymbol{Y}$, consisting of phantom mappings $h_\mu\colon Z \to Y_\mu$?