Univerza v Mariboru

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Fakulteta za naravoslovje in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	Osnove teorije mere
Course title:	Basic measure theory

Študijski program in stopnja **Study programme and level**	**Študijska smer** **Study field**	**Letnik** **Academic year**	**Semester** **Semester**
Matematika 2. stopnja		1. ali 2.	1. ali 3.
Mathematics 2nd degree		1. or 2.	1. or 3.

Vrsta predmeta / Course type

Univerzitetna koda predmeta / University course code:

Predavanja **Lectures**	**Seminar** **Seminar**	**Sem. vaje** **Tutorial**	**Lab. vaje** **Laboratory work**	**Teren. vaje** **Field work**	**Samost. delo** **Individ. work**	**ECTS**
45		30			135	**7**

Nosilec predmeta / Lecturer: Valerij Romanovskij

Jeziki / Languages:	
Predavanja / Lectures:	SLOVENSKO/SLOVENE
Vaje / Tutorial:	SLOVENSKO/SLOVENE

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:

Jih ni.

Prerequisits:

There are none.

Vsebina:

Content (Syllabus outline):

- Osnovni pojmi teorije mere: Algebra, σ-algebra, Borelova σ-algebra na R^n. Mere in osnovne lastnosti mer. Merljivi prostori. Pozitivne mere. Zunanje mere. Lebesqueova mera na R^n.
- Funkcije in integrali: Merljive funkcije. Stopničaste funkcije. Integral stopničaste funkcije. Integral merljive funkcije. Izrek o monotoni konvergenci. Fatoujeva lema in Lebesqueov izrek o dominantni konvergenci. Povezanost Riemannovega in Lebesqueovega integrala.
- Konvergenca: Zaporedja merljivih funkcij in konvergenca. Konvergenca skoraj povsod. Norma in normirani L^p-prostori. Neenakosti (Hölder, Minkowski). Dualni prostori.
- Predznačne in kompleksne mere: Predznačne mere in Hahnov razcepni izrek. Kompleksne mere in Radon-Nikodymov izrek. Funkcije z omejeno varianco.
- *Lebesgue-Stieltjesov integral.*

- Basic concepts of measure theory: Algebra, σ-algebra, Borel σ-algebra on R^n. Measure and its basic properties. Measurable spaces. Positive measures. Outer measures. Lebesque measure on R^n.
- Functions and integrals: Measurable functions. Simple measurable functions. The integral of a simple measurable function. The integral of a measurable function. The monotone convergence theorem. Fatou's lemma and Lebesque's dominated convergence theorem. Relationships between Riemann's and Lebesque's integral.
- Convergence: Sequences of measurable functions and convergence. Convergence almost everywhere. Norm and normed L^p-spaces. Inequalities (Hölder, Minkowski). Dual spaces.
- Signed and complex measures: Signed measures and the Hahn decomposition theorem. Complex measures and the Radon-Nikodym theorem. Functions of bounded variation.
- *Lebesgue-Stieltjes integral*

Temeljni literatura in viri / Readings:

1. M. Capinski, E. Kopp: *Measure, integral and probability*, Springer-Verlag London, 2004.
2. D. L. Cohn: *Measure theory,* Birkhäuser, 1994.
3. R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA, 2001.
4. M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA, 1985.
5. W. Rudin: *Real and complex analysis*, *3th edition*, Mc-Graw-Hill, 1986.
6. H. Sohrab, *Basic real analysis*, Birkhauser Boston, 2003.
7. I. Vidav, *Višja matematika II*, DZS, Ljubljana, 1975.

Cilji in kompetence:

Glavni cilj predmeta je proučiti temeljne koncepte in rezultate teorije mere.

Objectives and competences:

The main goal of the course is to study the fundamental concepts and results of measure theory.

Predvideni študijski rezultati:

Znanje in razumevanje:

- merljivi prostori, merljive funkcije, abstraktno integriranje, izreki o konvergenci, L^p-prostori, produktne mere, odvodi mer.

Prenesljive/ključne spretnosti in drugi atributi:

- Poznavanje osnov teorije mere je podlaga za študij različnih matematičnih področij (funkcionalne analize, verjetnosti, parcialnih diferencialnih enačb itd.).

Intended learning outcomes:

Knowledge and Understanding:

- Measurable spaces, measurable functions, abstract integration, convergence theorems, L^p-spaces, product measures, differentiation of measures.

Transferable/Key Skills and other attributes:

- Knowing the fundamentals of measure theory is a prerequisite for studying various mathematical areas (functional analysis, probability, partial differential equations etc.).

Metode poučevanja in učenja:	**Learning and teaching methods:**
• Predavanja • Teoretične vaje	• Lectures • Theoretical exercises

Načini ocenjevanja:	Delež (v %) / Weight (in %)	**Assessment:**
Pisni izpit	100%	Written exam

Reference nosilca / Lecturer's references:

1. BASOV, Vladimir V., ROMANOVSKI, Valery. Linearization of two-dimensional systems of ODEs without conditions on small denominators. *Appl. math. lett.*. [Print ed.], 2012, vol. 25, iss. 2, str. 99-103. http://dx.doi.org/10.1016/j.aml.2011.06.029, doi: 10.1016/j.aml.2011.06.029. [COBISS.SI-ID 18675208]

2. LEVANDOVSKYY, Viktor, PFISTER, Gerhard, ROMANOVSKI, Valery. Evaluating cyclicity of cubic systems with algorithms of computational algebra. *Commun. pure appl. anal.*, 2012, vol. 11, no. 5, str. 2023-2035, doi: 10.3934/cpaa.2012.11.2023. [COBISS.SI-ID 19075080]

3. WENTAO, Huang, CHEN, Xingwu, ROMANOVSKI, Valery. Linear centers with perturbations of degree 2d + 5. *Int. j. bifurc. chaos appl. sci. eng.*, 2012, vol. 22, no. 1, str. [1250007-1 - 1250007-12]. http://www.ejournals.wspc.com.sg/ijbc/22/2201/S0218127412500071.html, doi: 10.1142/S0218127412500071. [COBISS.SI-ID 69213185]

4. EDNERAL, Victor F., MAHDI, Adam, ROMANOVSKI, Valery, SHAFER, Douglas. The center problem on a center manifold in [R] [sup] 3. *Nonlinear anal.*. [Print ed.], 2012, vol. 75, no. 4, str. 2614-2622. http://dx.doi.org/10.1016/j.na.2011.11.006. [COBISS.SI-ID 18891016]

5. LIANG, Feng, HAN, Maoan, ROMANOVSKI, Valery. Bifurcation of limit cycles by perturbing a piecewise linear Hamiltonian system with a homoclinic loop. *Nonlinear anal.*. [Print ed.], 2012, vol. 75, no. 11, str. 4355-4374. http://dx.doi.org/10.1016/j.na.2012.03.022. [COBISS.SI-ID 19146760]